Application of modern tools for the thermo-acoustic study of annular combustion chambers

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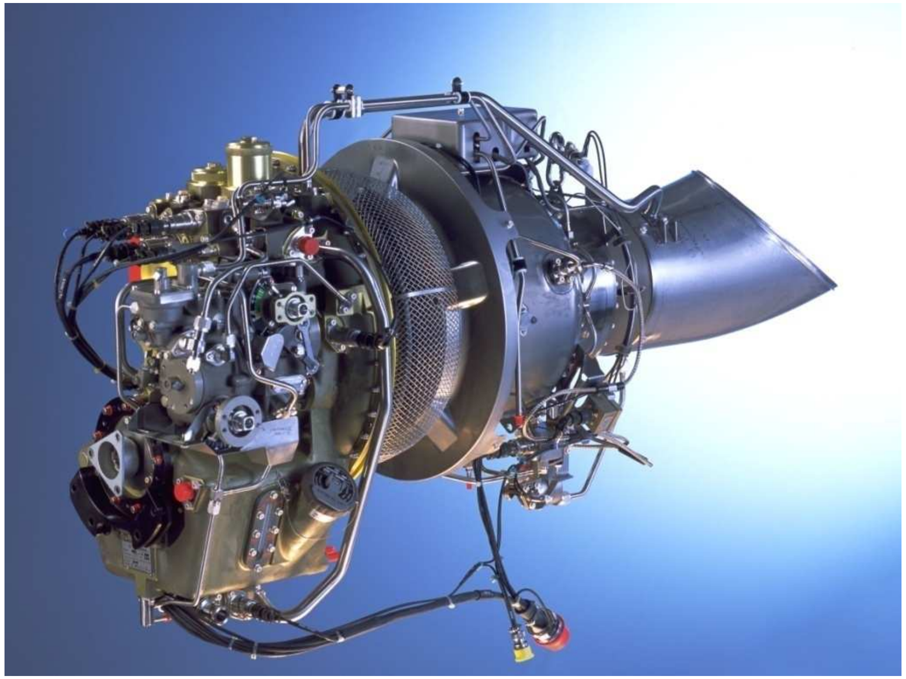
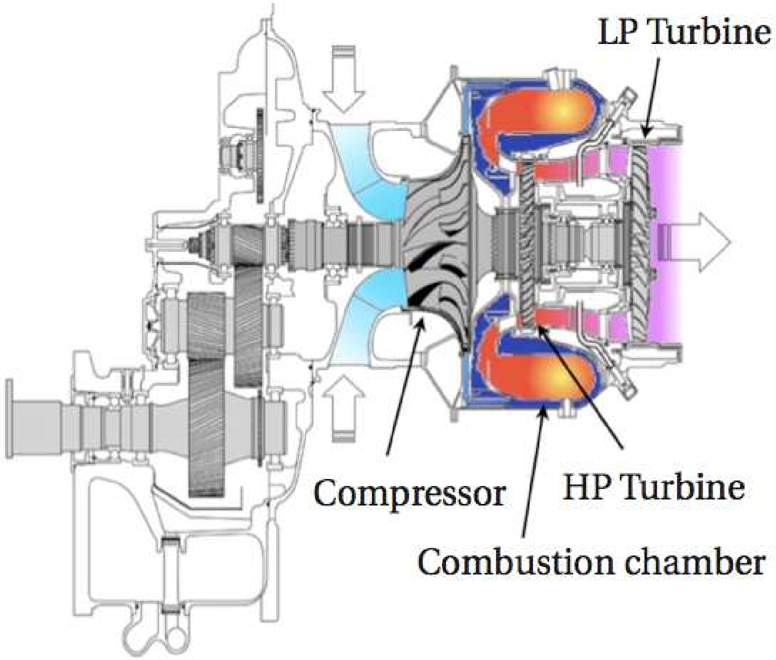
and

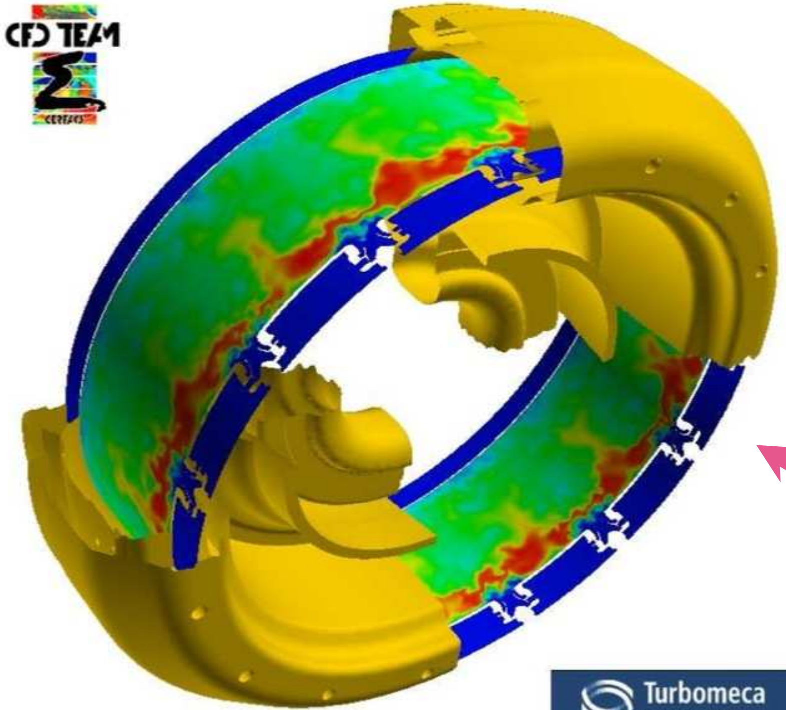
CERFACS

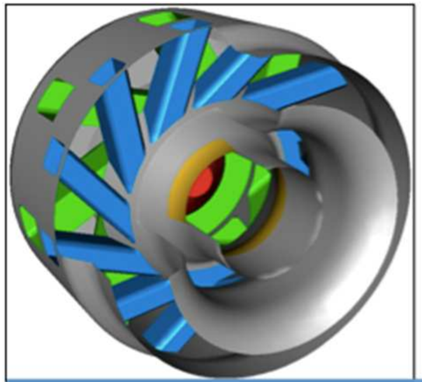
Introduction





Introduction

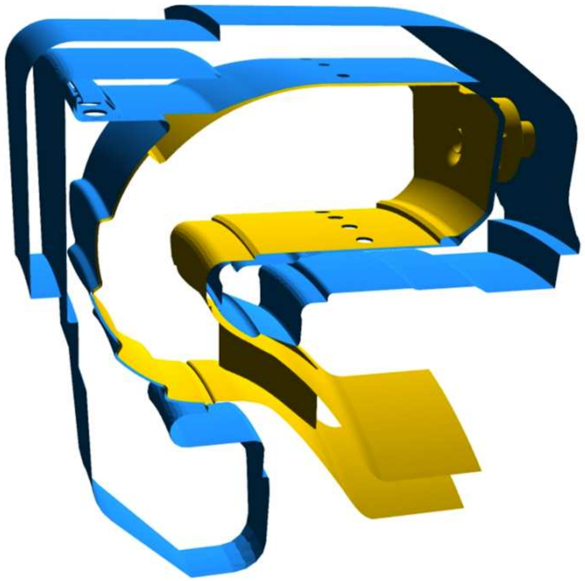




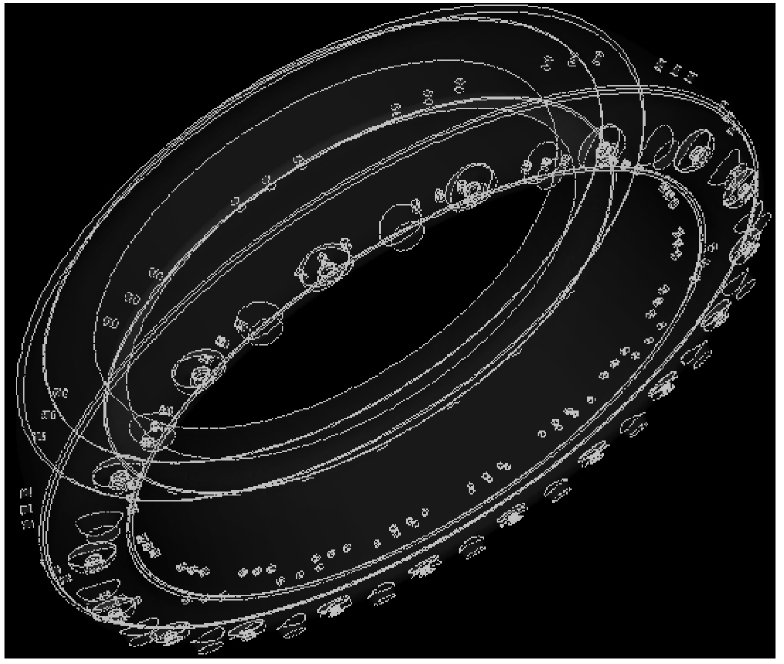
LPP injector

One swirler per sector

10-24 sectors

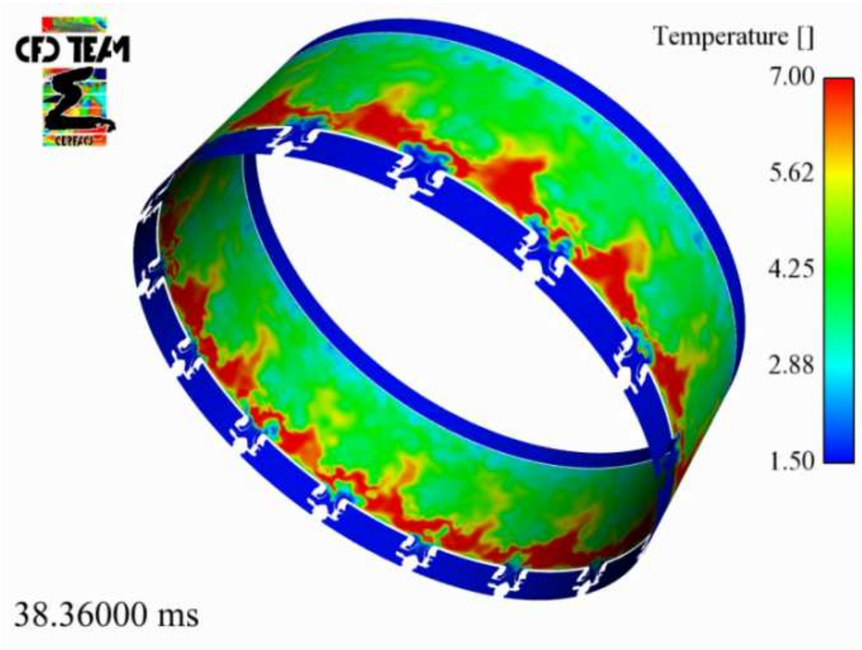


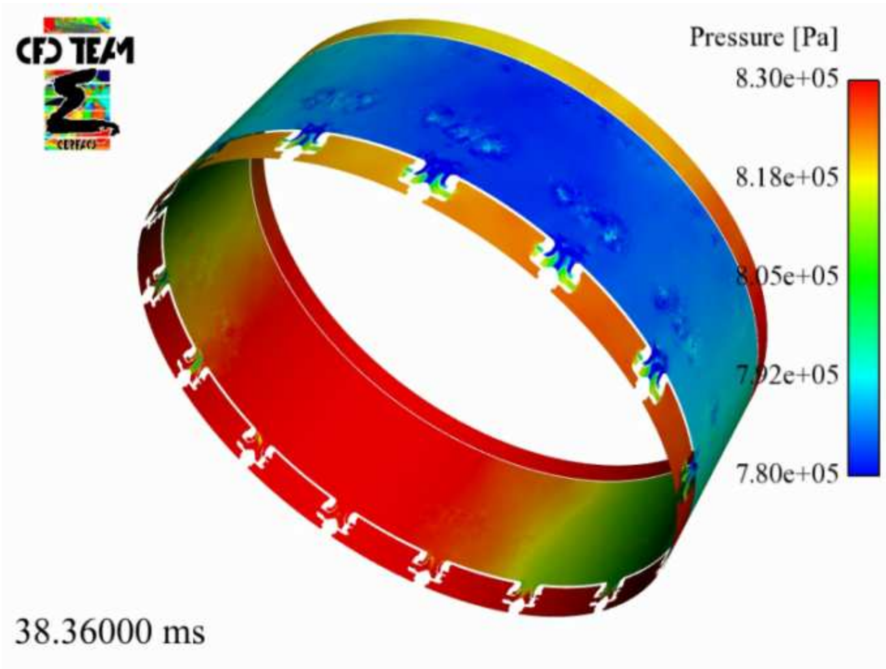
Introduction



Y. Sommerer & M. Boileau
CERFACS

Introduction





SOME KEY INGREDIENTS

- Flow physics
  - turbulence, partial mixing, chemistry, two-phase flow , combustion modeling, heat loss, wall treatment, radiative transfer, …
- Acoustics
  - complex impedance, mean flow effects, acoustics/flame coupling, non-linearity, limit cycle, non-normality, mode interactions, …
- Numerics
  - Low dispersive – low dissipative schemes, non linear stability, scalability, non-linear eigen value problems, …

PARALLEL COMPUTING

- www.top500.org – june 2010

#	Site	Computer
1	Oak Ridge National Laboratory USA	Cray XT5-HE Opteron Six Core 2.6 GHz
2	National Supercomputing Centre in Shenzhen (NSCS) **China**	Dawning TC3600 Blade, Intel X5650, NVidia Tesla C2050 GPU
3	DOE/NNSA/LANL USA	BladeCenter QS22/LS21 Cluster, PowerXCell 8i 3.2 Ghz / Opteron DC 1.8 GHz, Voltaire Infiniband
4	National Institute for Computational Sciences USA	Cray XT5-HE Opteron Six Core 2.6 GHz
5	Forschungszentrum Juelich (FZJ) **Germany**	Blue Gene/P Solution
6	NASA/Ames Research Center/NAS USA	SGI Altix ICE 8200EX/8400EX, Xeon HT QC 3.0/Xeon Westmere 2.93 Ghz, Infiniband
7	National SuperComputer Center in Tianjin/NUDT **China**	NUDT TH-1 Cluster, Xeon E5540/E5450, ATI Radeon HD 4870 2, Infiniband
8	DOE/NNSA/LLNL USA	eServer Blue Gene Solution
9	Argonne National Laboratory USA	Blue Gene/P Solution
10	Sandia National Laboratories USA	Cray XT5-HE Opteron Six Core 2.6 GHz

70 000 processors or more
458 Tflops or more

PARALLEL COMPUTING

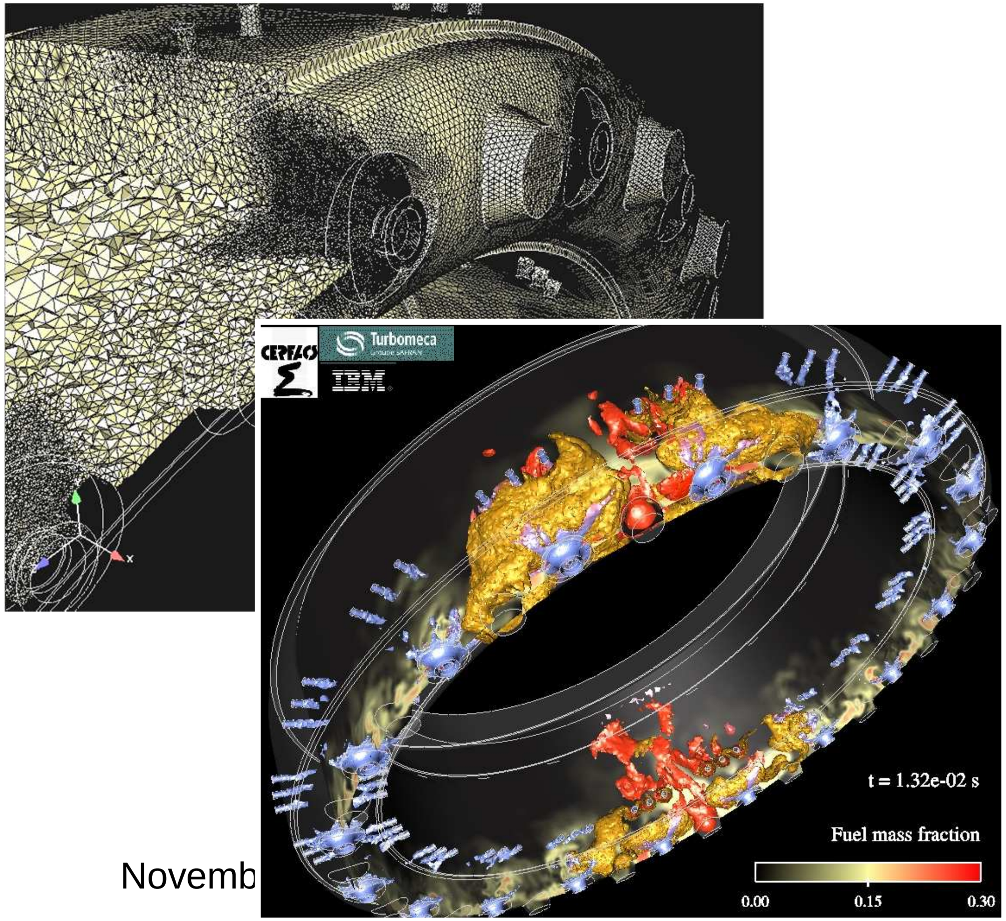
- www.top500.org – june 2007 **(3 years ago …)**

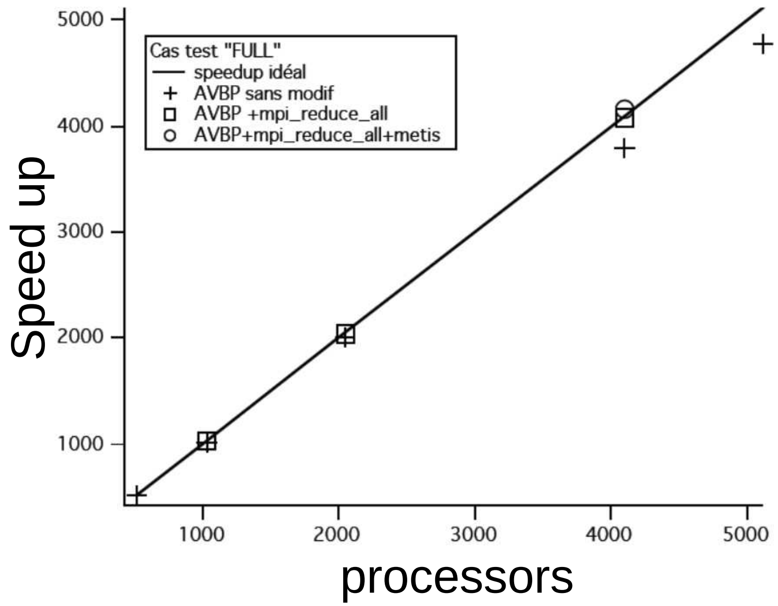
#	Site	Computer
1	DOE/NNSA/LLNL United States	BlueGene/L - eServer Blue Gene Solution IBM
2	Oak Ridge National Laboratory United States	Jaguar - Cray XT4/XT3 Cray Inc.
3	NNSA/Sandia National Laboratories United States	Red Storm - Sandia/ Cray Red Storm, Cray Inc.
4	IBM Thomas J. Watson Research Center United States	BGW - eServer Blue Gene Solution IBM
5	Stony Brook/BNL, New York Center for Computional United States	New York Blue - eServer Blue Gene Solution IBM
6	DOE/NNSA/LLNL United States	ASC Purple - eServer pSeries p5 575 1.9 GHz IBM
7	Rensselaer Polytechnic Institute, Computional Center United States	eServer Blue Gene Solution IBM
8	NCSA United States	Abe - PowerEdge 1955, 2.33 GHz, Infiniband Dell
9	Barcelona Supercomputing Center **Spain**	MareNostrum - BladeCenter JS21 Cluster, IBM
10	Leibniz Rechenzentrum **Germany**	HLRB-II - Altix 4700 1.6 GHz SGI

10 000 processors or more **(factor 7)**
56 Tflops or more **(factor 8)**

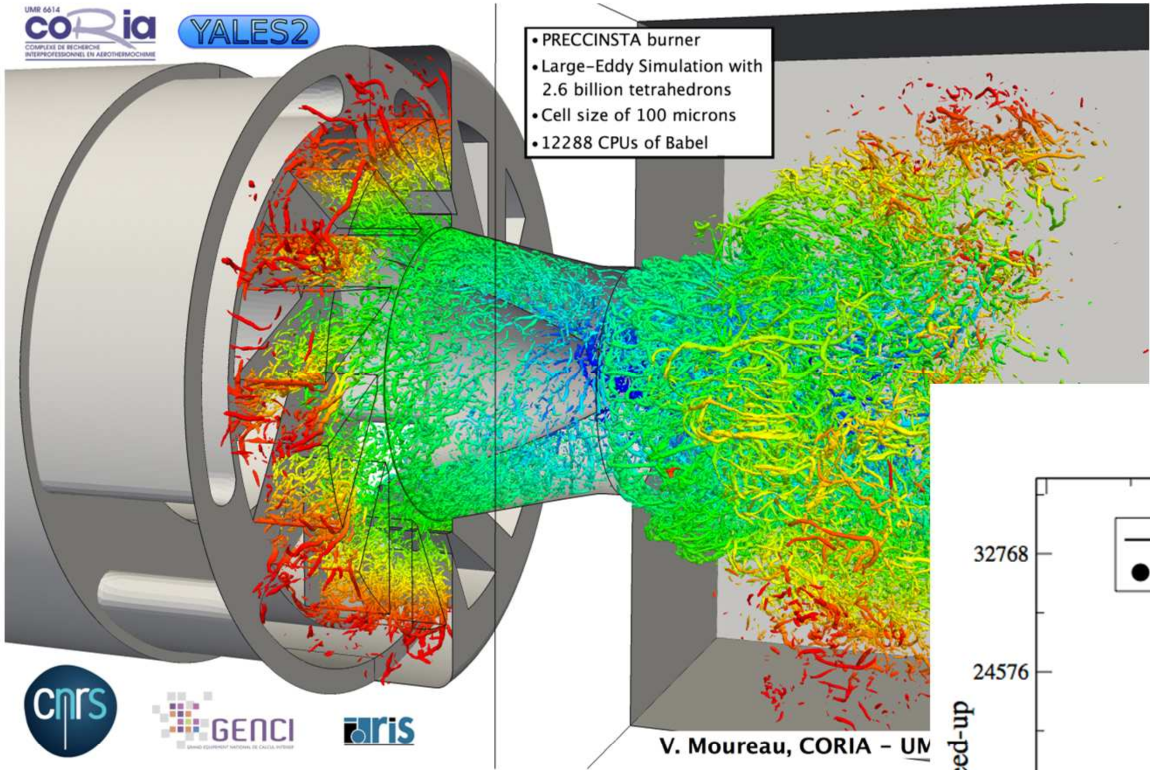
PARALLEL COMPUTING

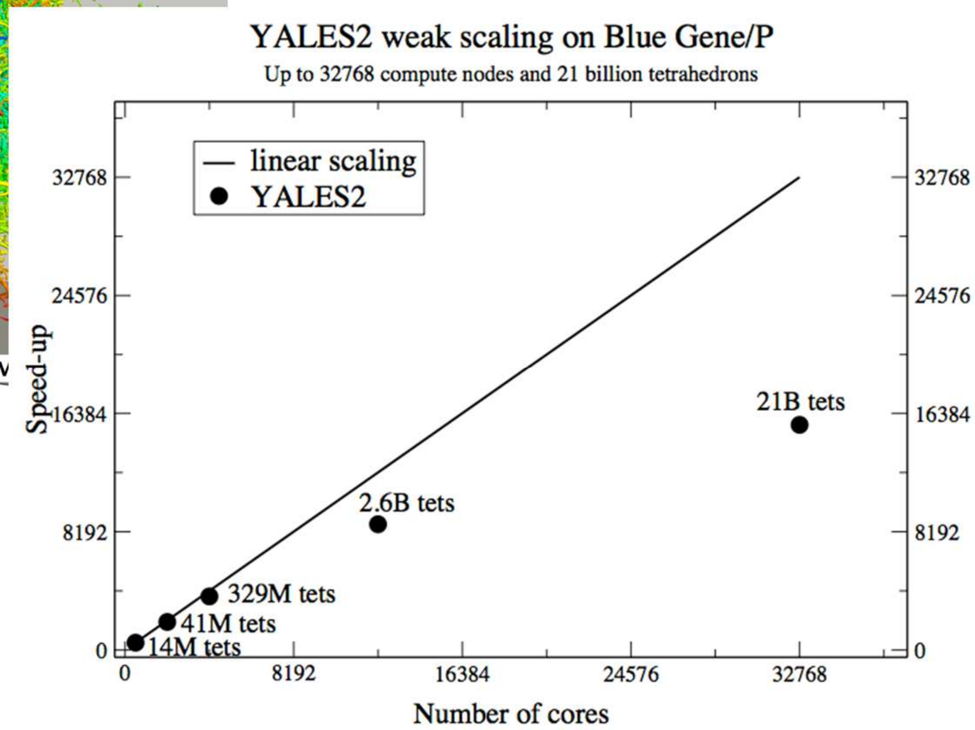
- Large scale unsteady computations require huge computing resources, an efficient codes …





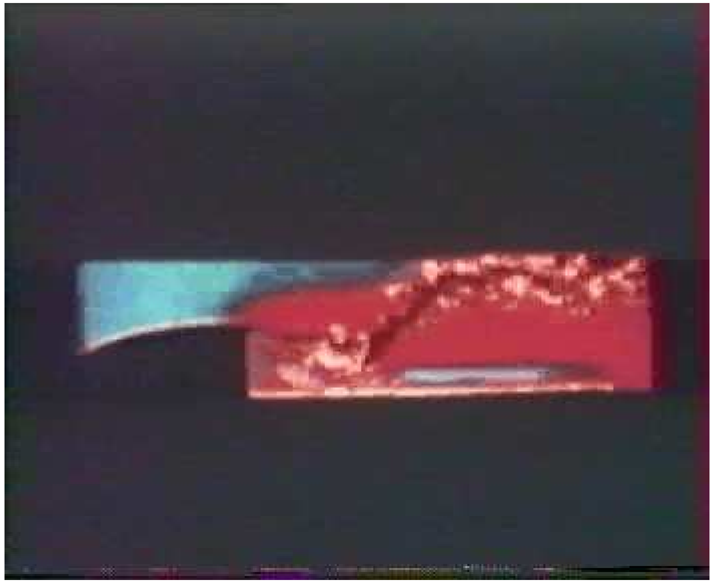
PARALLEL COMPUTING





Thermo-acoustic instabilities

Premixed gas →



The Berkeley backward facing step experiment.

Thermo-acoustic instabilities

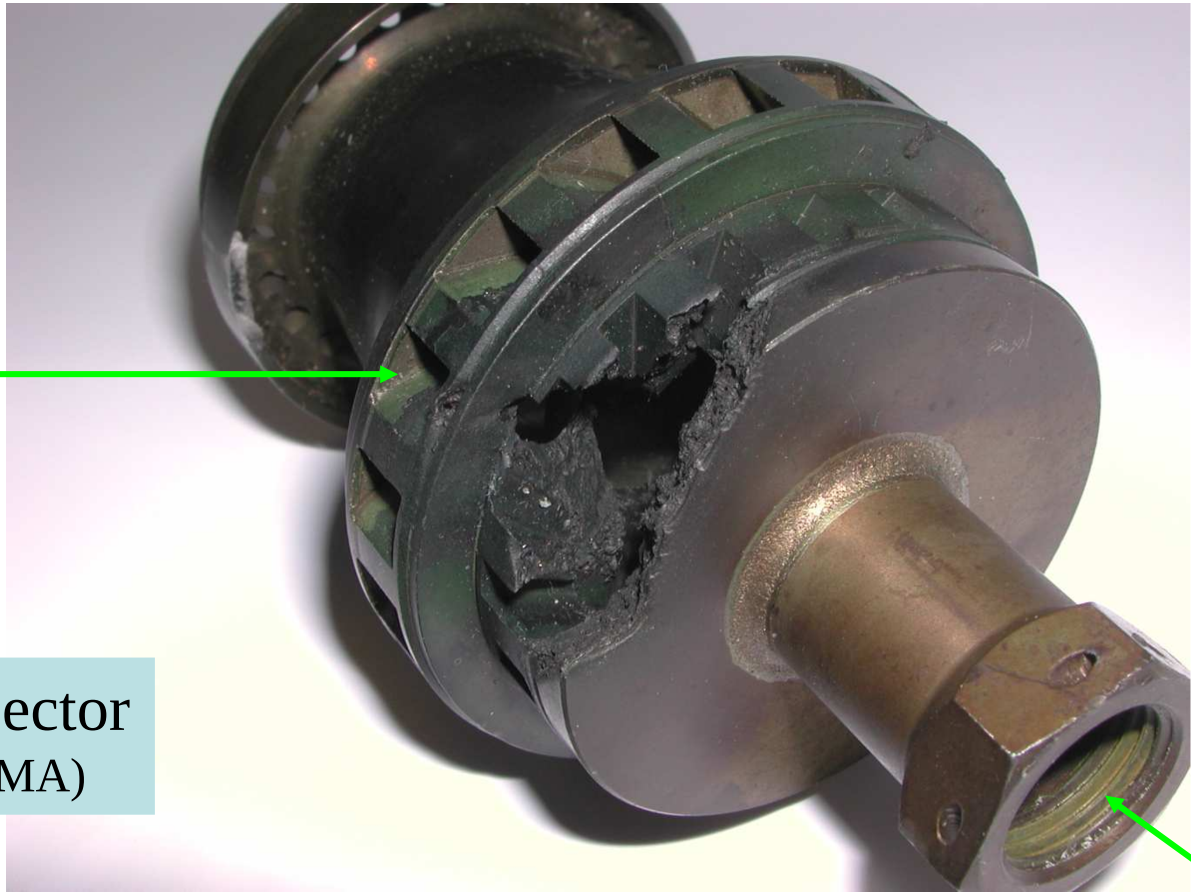
- Self-sustained oscillations arising from the coupling between a source of heat and the acoustic waves of the system

- Known since a very long time (Rijke, 1859; Rayleigh, 1878)

- Not fully understood yet …

- but surely not desirable …

Better avoid them …

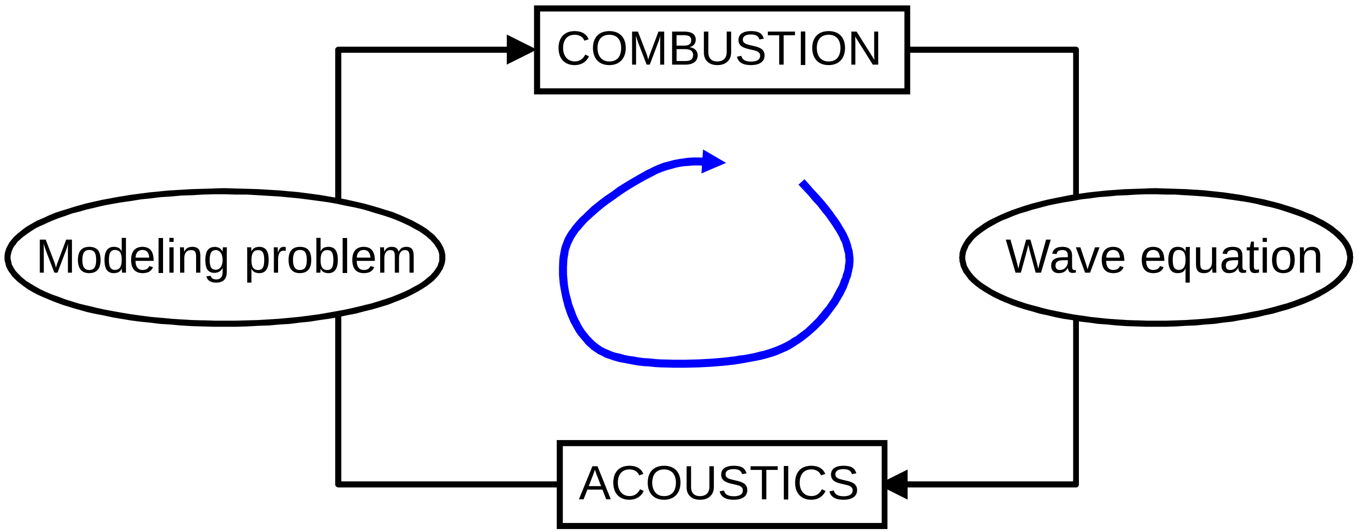


AIR

LPP injector
(SNECMA)

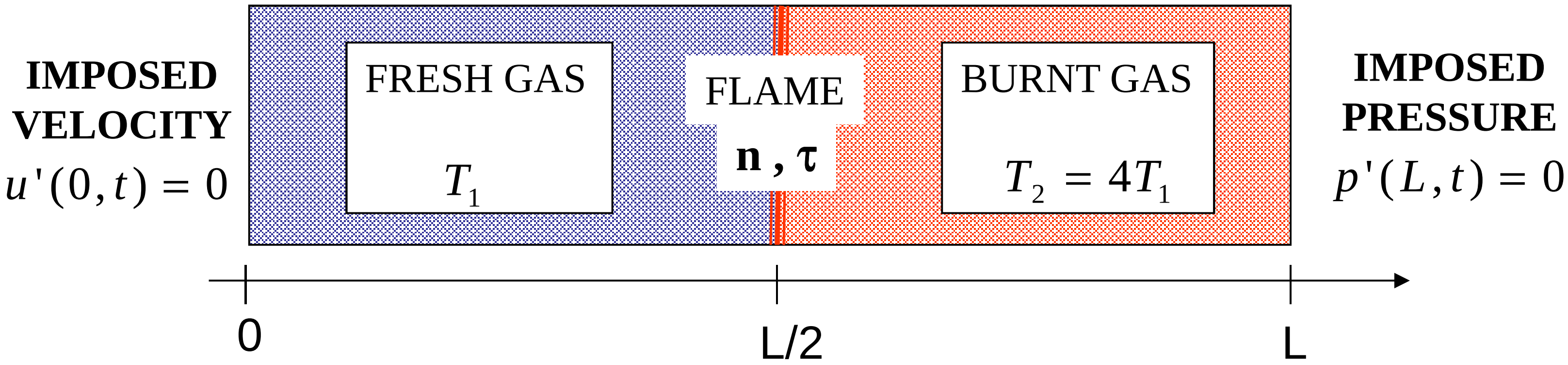
FUEL

Flame/acoustics coupling



Rayleigh criterion:
Flame/acoustics coupling promotes instability if pressure and heat release fluctuations are in phase

A tractable 1D problem



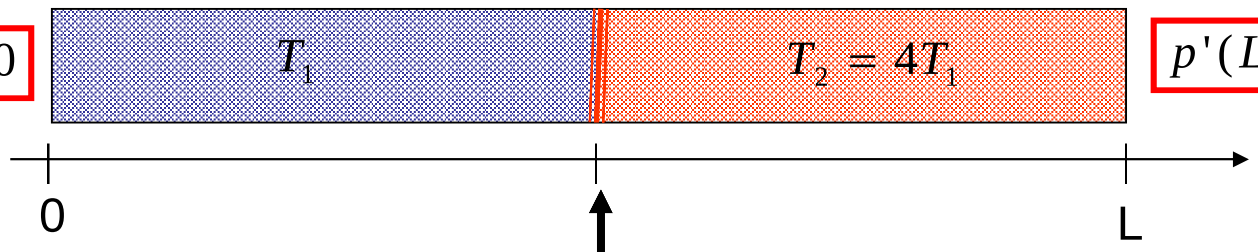
Kaufmann, Nicoud & Poinsot, *Comb. Flame*, 2002

$$q'(x,t) = \frac{\gamma p_0}{\gamma - 1} \times n \times u'(L/2^-, t-\tau) \times \delta\left(x - \frac{L}{2}\right)$$

$n-\tau$ model for the unsteady heat release $\begin{cases} \text{n : interaction index} \\ \tau : \text{time lag} \end{cases}$

Equations

$u'(0,t) = 0$



$p'(L,t) = 0$

$$0 < x < \frac{L}{2}:$$

$$\frac{\partial^2 p'}{\partial t^2} - c_1^2 \frac{\partial^2 p'}{\partial x^2} = 0$$

CLASSICAL ACOUSTICS
2 wave amplitudes

$$\frac{L}{2} < x < L:$$

$$\frac{\partial^2 p'}{\partial t^2} - c_2^2 \frac{\partial^2 p'}{\partial x^2} = 0$$

CLASSICAL ACOUSTICS
2 wave amplitudes

$$\overbrace{\left[p'\right]_{L/2^-}^{L/2^+} = 0 \quad \text{and} \quad \left[u'\right]_{L/2^-}^{L/2^+} = n \times u'\left(L^- / 2, t - \tau\right)}^{\text{TWO JUMP RELATIONS}}$$

Dispersion relation

- Solve the 4x4 homogeneous linear system to find out the 4 wave amplitudes

- Consider Fourier modes

$$p'(x,t) = \Re\left(\hat{p}(x)e^{-j\omega t}\right) \qquad \begin{cases} \Im(\omega) < 0: \text{ damped mode} \\ \Im(\omega) > 0: \text{ amplified mode} \end{cases}$$

- Condition for non-trivial (zero) solutions to exist

$$\underbrace{\cos\left(\frac{\omega L}{4c_1}\right)}_{\text{Uncoupled modes}} \times \underbrace{\left[\cos^2\left(\frac{\omega L}{4c_1}\right) - \frac{3}{4} - \frac{1}{4}\frac{ne^{j\omega\tau} - 1}{ne^{j\omega\tau} + 3}\right]}_{\text{Coupled modes}} = 0$$

Uncoupled modes **Coupled modes**

Stability of the coupled modes

- Eigen frequencies

$$\cos^2\left(\frac{\omega L}{4c_1}\right) - \frac{3}{4} - \frac{1}{4}\frac{ne^{j\omega\tau} - 1}{ne^{j\omega\tau} + 3} = 0$$

- Steady flame *n=0*:

$$\omega_{m,0} = \frac{4c_1}{L}\left[\pm \arccos\left(\pm\sqrt{\frac{2}{3}}\right) + 2m\pi\right], \quad m = 0,1,2,...$$

- Asymptotic development for *n<<1*:

$$\omega_m = \omega_{m,0} \underbrace{- n\frac{4c_1}{9L\sin\left(\omega_{m,0}L/2c_1\right)}\left[\cos\left(\omega_{m,0}\tau\right) + j\sin\left(\omega_{m,0}\tau\right)\right]}_{\text{Complex pulsation shift}} + o(n)$$

Kaufmann, Nicoud & Poinsot, *Comb. Flame*, 2002

Time lag effect

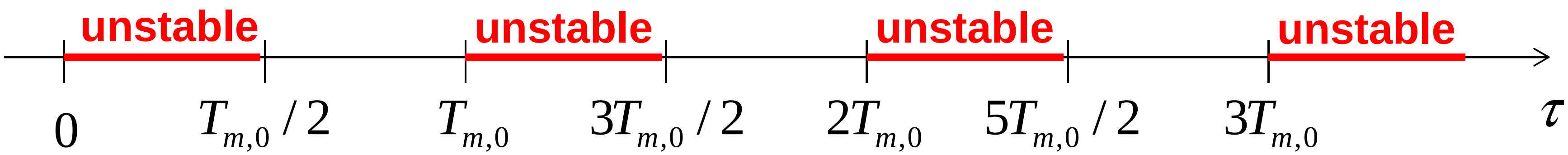
- The imaginary part of the frequency is

$$-n\frac{4c_1\sin(\omega_{m,0}\tau)}{9L\sin(\omega_{m,0}L/2c_1)}$$

- Steady flame modes such that $\sin(\omega_{m,0}L/2c_1)<0$

- The unsteady HR destabilizes the flame if

$$\sin(\omega_{m,0}\tau)>0 \quad\Rightarrow\quad 0<\omega_{m,0}\tau<\pi[2\pi] \quad\Rightarrow\quad 0<\tau<\frac{T_{m,0}}{2}\,[T_{m,0}]$$



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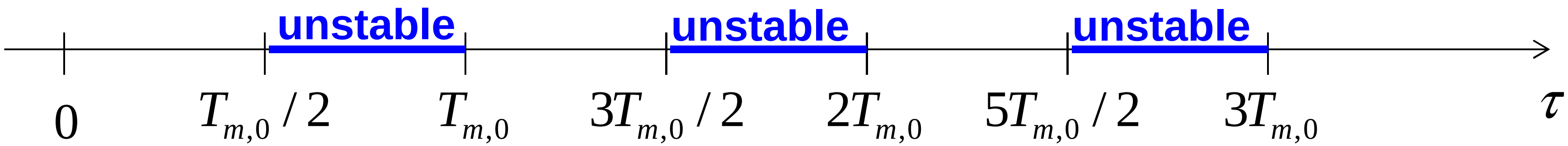
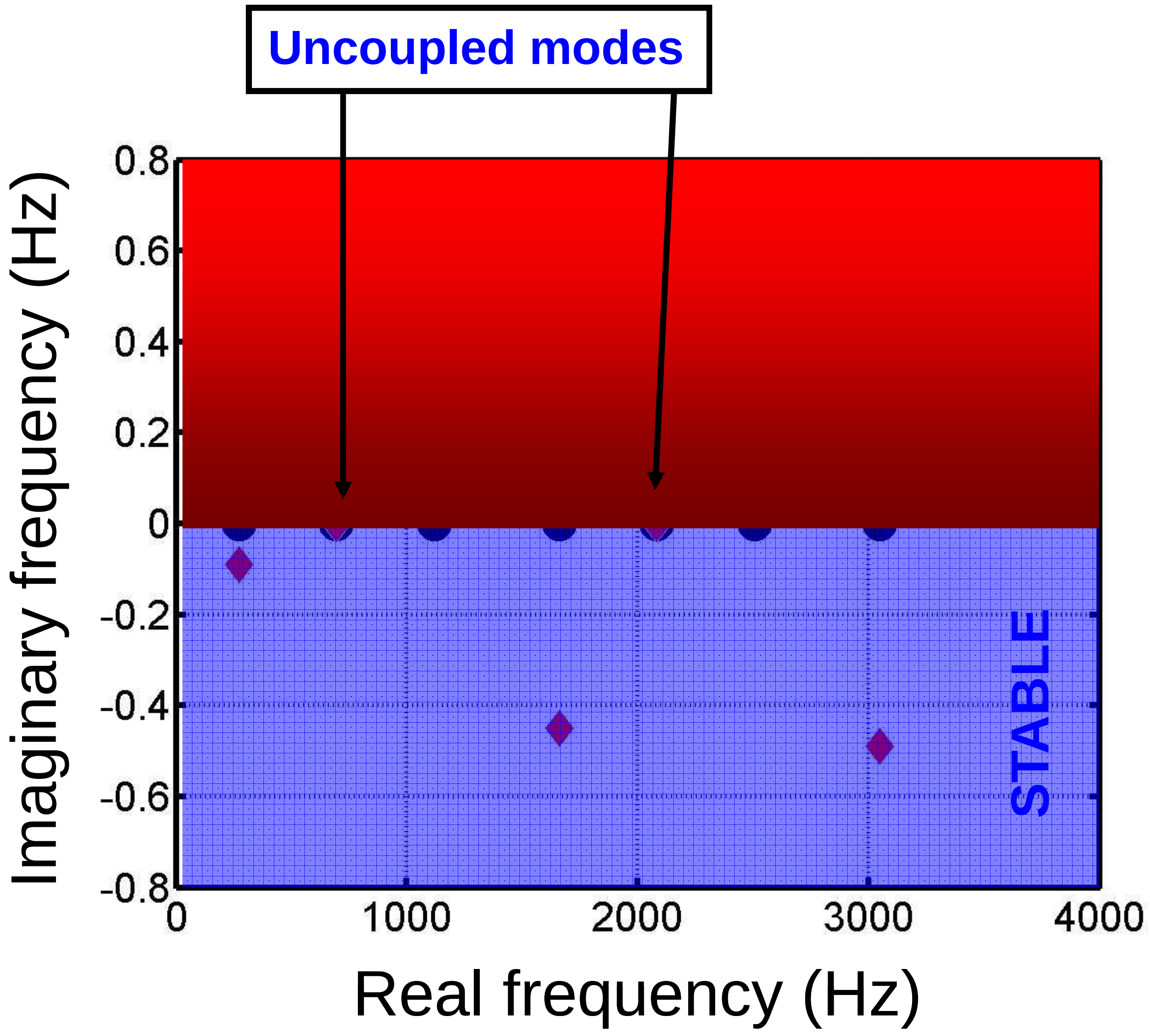


Numerical example

$$T_1 = 300 \text{ K}$$
$$L = 0.5 \text{ m}$$

● **Steady flame**

◆ **Unteady flame**
n=0.01
τ=0.1 ms



OUTLINE

1. Computing the whole flow
2. Computing the fluctuations
3. Boundary conditions
4. Analysis of an annular combustor

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1. Computing the whole flow
2. Computing the fluctuations
3. Boundary conditions
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BASIC EQUATIONS

reacting, multi-species gaseous mixture

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u_i)}{\partial x_i} = 0$$

$$\frac{\partial (\rho Y_k)}{\partial t} + \frac{\partial}{\partial x_i} \left(\rho \left(u_i + V_{k,i}\right) Y_k\right) = \dot{\omega}_k$$

$$\frac{\partial(\rho u_i)}{\partial t} + \frac{\partial(\rho u_i u_j)}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{\partial \tau_{ij}}{\partial x_j}$$

$$\rho \frac{DE}{Dt} = -\frac{\partial q_i^*}{\partial x_i} + \frac{\partial}{\partial x_j}(\tau_{ij} u_i) - \frac{\partial}{\partial x_i}(p u_i) + \dot{\omega}_T$$

$$\frac{p}{\rho} = r\, T$$

$$r = R/W$$

$$W = \sum_k X_k W_k$$

$$Y_k = X_k W_k / W$$

$$\dot{\omega}_T = -\sum_k \Delta h^0_{f,k} \dot{\omega}_k$$

$$C_p = \sum_k C_{p,k} Y_k$$

BASIC EQUATIONS
energy / enthalpy forms

$h_{s,k} = \int_{T_0}^{T} C_{p,k}\, dT$	Sensible enthalpy of species k
$h_k = \int_{T_0}^{T} C_{p,k}\, dT + \Delta h_{f,k}^0$	Specific enthalpy of species k
$h_s = \sum_k h_{s,k} Y_k$	Sensible enthalpy of the mixture
$h = h_s + \sum_k \Delta h_{f,k}^0 Y_k$	Specific enthalpy of the mixture
$h_t = h + u_i u_i / 2$	Total enthalpy of the mixture
$H = h_s + u_i u_i / 2$	Total non chemical enthalpy of the mixture
$E = H - p/\rho$	Total non chemical energy of the mixture

BASIC EQUATIONS

Diffusion velocity / mass flux

$$V_{k,i} = \underbrace{-\sum_l D_{kl} \frac{\partial X_l}{\partial x_i}}_{\text{mixture effect}} \underbrace{-\sum_l D_{kl} (X_l - Y_l) \frac{1}{p} \frac{\partial p}{\partial x_i}}_{\text{pressure gradient effect}} \underbrace{-\sum_l D_{kl} \, \chi_l \frac{\partial \ln T}{\partial x_i}}_{\text{Soret effect}}$$

Exact form

↓

$$\boxed{V_{k,i} = V_{k,i}^{hc} + V_i^{cor}}$$

Practical model

$$V_{k,i}^{hc} X_k = -\mathcal{D}_k \frac{\partial X_k}{\partial x_i}$$

$$V_i^{cor} = \sum_k \mathcal{D}_k \frac{W_k}{W} \frac{\partial X_k}{\partial x_i}$$

$$\boxed{\sum_k Y_k V_{k,i} = 0}$$

← Satisfies mass conservation

BASIC EQUATIONS

Stress and heat flux

$$\tau_{ij} = -\frac{2}{3}\mu\frac{\partial u_l}{\partial x_l}\,\delta_{ij} + \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

$$q_i = \underbrace{-\lambda\frac{\partial T}{\partial x_i} + \rho\sum_k h_{s,k}Y_kV_{k,i}}_{q_i^*} + \rho\sum_k \Delta h^0_{f,k}Y_kV_{k,i}$$

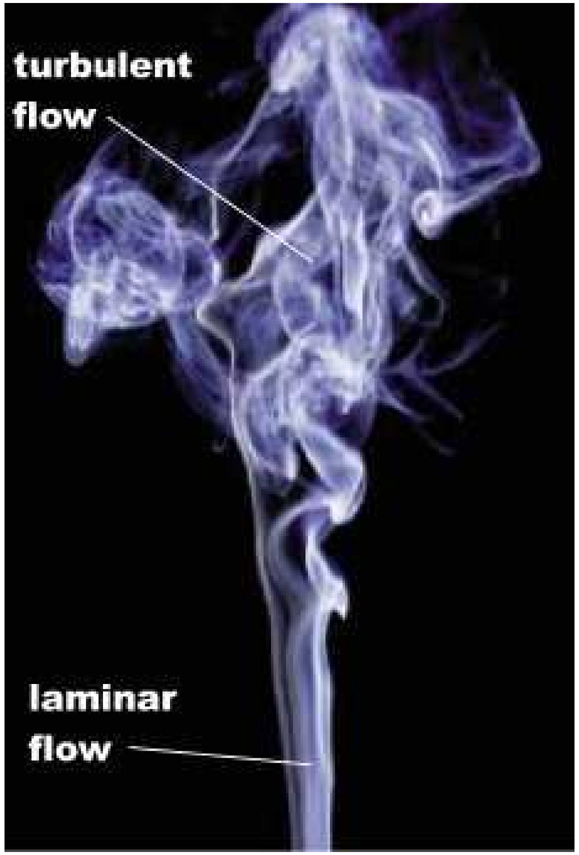
$$\mu = \mu_{ref}\left(\frac{T}{T_{ref}}\right)^c$$

$$\lambda = \frac{\mu C_p}{\mathrm{Pr}}$$

$$\mathcal{D}_k = \frac{1 - Y_k}{\sum_{j\neq k} X_j/\mathcal{D}_{jk}}$$

$$\mathcal{D}_k = \nu/Sc_k.$$

Turbulence



1. Turbulence is contained in the NS equations
2. The flow regime (laminar vs turbulent) depends on the Reynolds number :

Velocity — Length scale

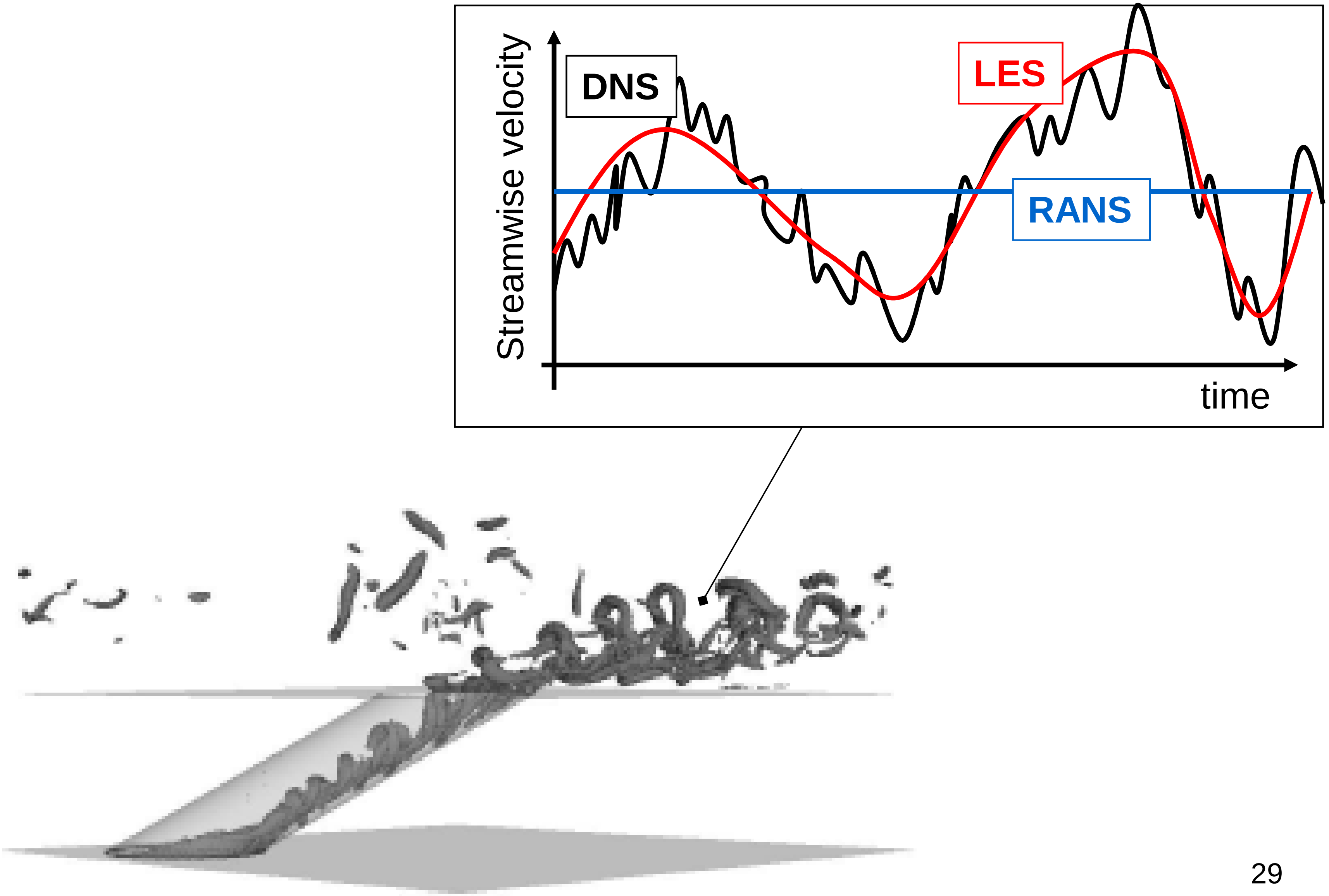
$$\mathrm{Re} = \frac{Ud}{\nu}$$

ν — viscosity

laminar → turbulent

Re

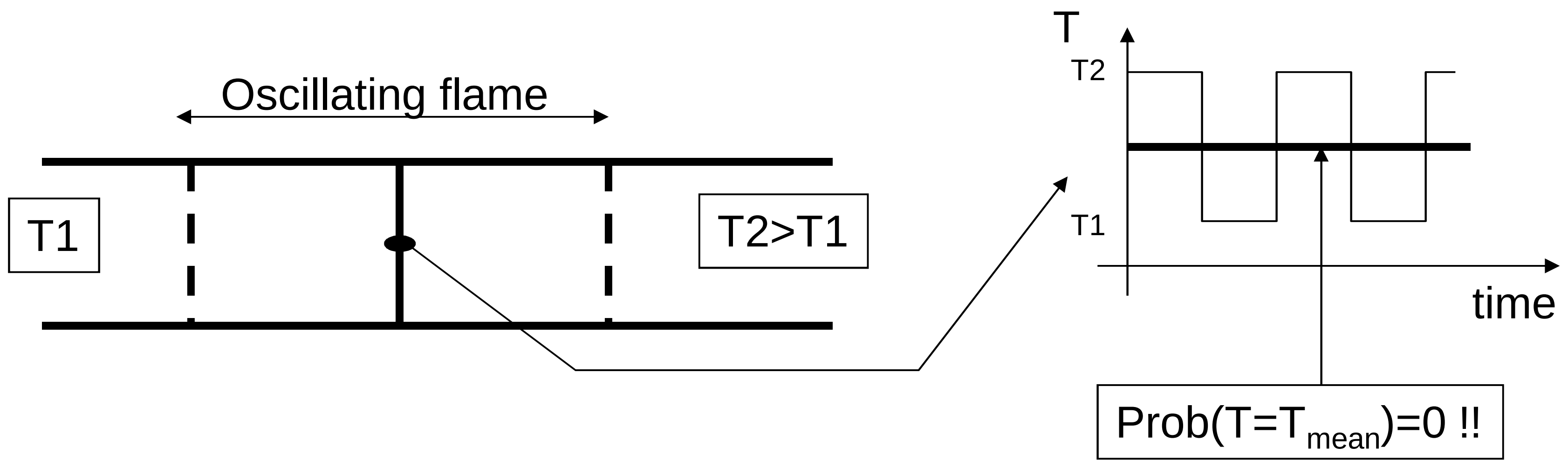
RANS – LES - DNS



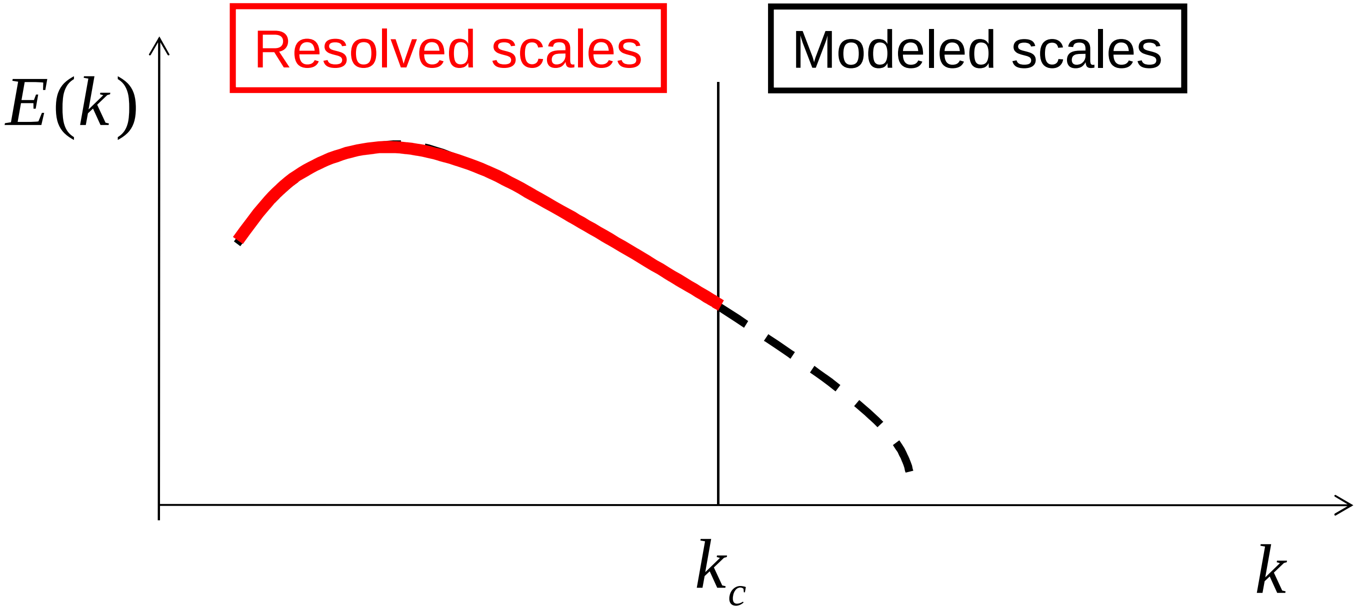
About the RANS approach

- Averages are not always enough (instabilities, growth rate, vortex shedding)

- Averages are even not always meaningful



The basic idea of LES



$$\overline{f(\mathbf{x},t)} = \int_{-\infty}^{+\infty} f(\mathbf{x}',t)\, G(\mathbf{x}'-\mathbf{x})\, d\mathbf{x}'$$

$$\widetilde{f(\mathbf{x},t)} = \frac{1}{\overline{\rho(\mathbf{x},t)}} \int_{-\infty}^{+\infty} \rho(\mathbf{x}',t)\, f(\mathbf{x}',t)\, G(\mathbf{x}'-\mathbf{x})\, d\mathbf{x}'.$$

LES equations

- Assumes small commutation errors
- Filtered version of the flow equations :

$$\frac{\partial \overline{\rho}\,\widetilde{Y_k}}{\partial t} + \frac{\partial}{\partial x_j}(\overline{\rho}\,\widetilde{Y_k}\,\widetilde{u_j}) = -\frac{\partial}{\partial x_j}[\overline{J_{k,j}} + J_{k,j}^{\mathrm{SGS}}] + \overline{\dot{\omega}_k},$$

$$\frac{\partial \overline{\rho}\,\widetilde{u_i}}{\partial t} + \frac{\partial}{\partial x_j}(\overline{\rho}\,\widetilde{u_i}\,\widetilde{u_j}) = -\frac{\partial}{\partial x_j}[\overline{P}\,\delta_{ij} - \overline{\tau_{ij}} - \tau_{ij}^{\mathrm{SGS}}],$$

$$\frac{\partial \overline{\rho}\,\widetilde{E}}{\partial t} + \frac{\partial}{\partial x_j}(\overline{\rho}\,\widetilde{E}\,\widetilde{u_j}) = -\frac{\partial}{\partial x_j}[\overline{u_i\,(P\,\delta_{ij} - \tau_{ij})} + \overline{q_j^*} + q_j^{\mathrm{SGS}}] + \overline{\dot{\omega}_T}.$$

Laminar contributions

- Assumes negligible cross correlation between gradient and diffusion coefficients:

$$\begin{aligned}\overline{J_{k,i}} &= -\overline{\rho\left(D_k \frac{W_k}{W}\frac{\partial X_k}{\partial x_i} - Y_k V_i^{cor}\right)} \\ &\approx -\overline{\rho}\left(\overline{D}_k \frac{W_k}{W}\frac{\partial \widetilde{X}_k}{\partial x_i} - \widetilde{Y}_k \widetilde{V}_i^{cor}\right)\end{aligned}$$

$$\begin{aligned}\overline{q_i^*} &= -\overline{\lambda \frac{\partial T}{\partial x_i}} + \sum_{k=1}^{N} \overline{J_{k,i} h_{s,k}} \\ &\approx -\overline{\lambda}\frac{\partial \widetilde{T}}{\partial x_i} + \sum_{k=1}^{N} \overline{J_{k,i}}\, \widetilde{h}_{s,k}\end{aligned}$$

$$\begin{aligned}\overline{\tau_{ij}} &= \overline{2\mu(S_{ij} - \tfrac{1}{3}\delta_{ij}S_{ll})} \\ &\approx 2\overline{\mu}(\widetilde{S}_{ij} - \tfrac{1}{3}\delta_{ij}\widetilde{S}_{ll})\end{aligned}$$

Sub-grid scale contributions

- Sub-grid scale stress tensor to be modeled

$$\tau_{ij}^{sgs} = \bar{\rho}\,\tilde{u}_i\tilde{u}_j - \overline{\rho\, u_i u_j}$$

- Sub-grid scale mass flux to be modeled

$$J_{k,j}^{sgs} = -\bar{\rho}\,\tilde{Y}_k\tilde{u}_j + \overline{\rho\, Y_k u_j}$$

- Sub-grid scale heat flux to be modeled

$$q_j^{sgs} = -\bar{\rho}\,\tilde{E}\,\tilde{u}_j + \overline{\rho\, E\, u_j}$$

The Smagorinsky model

- From dimensional consideration, simply assume:

$$\tau_{ij}^{sgs} - \frac{1}{3}\tau_{kk}^{sgs}\delta_{ij} = 2\mu_{sgs}\left(\tilde{S}_{ij} - \frac{1}{3}\tilde{S}_{kk}\delta_{ij}\right), \quad \text{with} \quad \tilde{S}_{ij} = \frac{1}{2}\left(\frac{\partial \tilde{u}_i}{\partial x_j} + \frac{\partial \tilde{u}_j}{\partial x_i}\right)$$

$$\boxed{\mu_{sgs} = \bar{\rho}(C_s\Delta)^2\sqrt{2\tilde{S}_{ij}\,\tilde{S}_{ij}}}$$

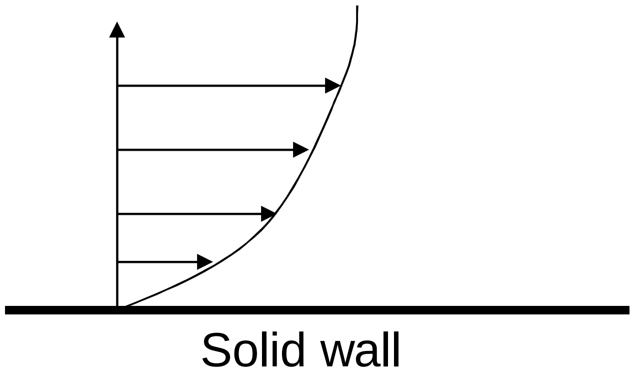
- The Smagorinsky constant is fixed so that the proper dissipation rate is produced, $C_s = 0.18$

The Smagorinsky model

- The sgs dissipation is positive $\varepsilon_{sgs} = \tau_{ij}^{sgs}\tilde{S}_{ij} \approx 2\mu_{sgs}\tilde{S}_{ij}\tilde{S}_{ij}$ always

- Very simple to implement, no extra CPU time

- Any mean gradient induces sub-grid scale activity and dissipation, even in 2D !!



$$V = W = 0 \text{ but } \mu_{sgs} \neq 0$$

$$\text{because } U = U(y) \text{ and } S_{12} \neq 0$$

No laminar-to-turbulent transition possible

- Strong limitation due to its lack of universality.
  Eg.: in a channel flow, Cs=0.1 should be used

The Dynamic procedure (constant ρ)

- By performing $\overline{NS}$, the following sgs contribution appears

$$\rho\frac{\partial \overline{u}_i}{\partial t}+\rho\frac{\partial \overline{u}_i\,\overline{u}_j}{\partial x_j}=-\frac{\partial \overline{P}}{\partial x_i}+\frac{\partial\left(\overline{\tau}_{ij}+\tau_{ij}^{sgs}\right)}{\partial x_j} \qquad \tau_{ij}^{sgs}=\rho\overline{u}_i\,\overline{u}_j-\rho\overline{u_i u_j}$$

- Let's apply another filter to these equations

$$\rho\frac{\partial \overleftrightarrow{\overline{u}_i}}{\partial t}+\rho\frac{\partial \overleftrightarrow{\overline{u}_i\,\overline{u}_j}}{\partial x_j}=-\frac{\partial \overleftrightarrow{\overline{P}}}{\partial x_i}+\frac{\partial\left(\overleftrightarrow{\overline{\tau}_{ij}}+\overleftrightarrow{\tau_{ij}^{sgs}}\right)}{\partial x_j} \qquad \overleftrightarrow{\tau_{ij}^{sgs}}=\rho\overleftrightarrow{\overline{u}_i\,\overline{u}_j}-\rho\overleftrightarrow{\overline{u_i u_j}} \qquad \textbf{A}$$

- By performing $\overleftrightarrow{\overline{NS}}$, one obtains the following equations

$$\rho\frac{\partial \overleftrightarrow{\overline{u}_i}}{\partial t}+\rho\frac{\partial \overleftrightarrow{\overline{u}_i}\,\overleftrightarrow{\overline{u}_j}}{\partial x_j}=-\frac{\partial \overleftrightarrow{\overline{P}}}{\partial x_i}+\frac{\partial\left(\overleftrightarrow{\overline{\tau}_{ij}}+T_{ij}^{sgs}\right)}{\partial x_j} \qquad T_{ij}^{sgs}=\rho\overleftrightarrow{\overline{u}_i}\,\overleftrightarrow{\overline{u}_j}-\rho\overleftrightarrow{\overline{u_i u_j}} \qquad \textbf{B}$$

- From **A** and **B** one obtains

$$T_{ij}^{sgs}=\overleftrightarrow{\tau_{ij}^{sgs}}-\rho\overleftrightarrow{\overline{u}_i\,\overline{u}_j}+\rho\overleftrightarrow{\overline{u}_i}\,\overleftrightarrow{\overline{u}_j}$$

The dynamic Smagorinsky model

- Assume the Smagorinsky model is applied twice

$$\tau_{ij}^{sgs} - \frac{1}{3}\tau_{kk}^{sgs}\delta_{ij} = 2\rho\left(C_s\overline{\Delta}\right)^2 \sqrt{2\overline{S_{ij}}\;\overline{S_{ij}}}\;\overline{S_{ij}}$$

$$T_{ij}^{sgs} - \frac{1}{3}T_{kk}^{sgs}\delta_{ij} = 2\rho\left(C_s\overleftrightarrow{\overline{\Delta}}\right)^2 \sqrt{2\overleftrightarrow{\overline{S_{ij}}}\;\overleftrightarrow{\overline{S_{ij}}}}\;\overleftrightarrow{\overline{S_{ij}}}$$

- Assume the same constant can be used and write the Germano identity

$$\boxed{T_{ij}^{sgs} = \overleftrightarrow{\tau_{ij}^{sgs}} - \rho\overleftrightarrow{\overline{u_i}\;\overline{u_j}} + \rho\overleftrightarrow{\overline{u_i}}\;\overleftrightarrow{\overline{u_j}}}$$

↓

$$2\rho\left(C_s\overleftrightarrow{\overline{\Delta}}\right)^2 \sqrt{2\overleftrightarrow{\overline{S_{ij}}}\;\overleftrightarrow{\overline{S_{ij}}}}\;\overleftrightarrow{\overline{S_{ij}}} + \frac{1}{3}T_{kk}^{sgs}\delta_{ij} = 2\rho\left(C_s\overline{\Delta}\right)^2 \overleftrightarrow{\sqrt{2\overline{S_{ij}}\;\overline{S_{ij}}}\;\overline{S_{ij}}} + \frac{1}{3}\overleftrightarrow{\tau_{kk}^{sgs}}\delta_{ij} - \rho\overleftrightarrow{\overline{u_i}\;\overline{u_j}} + \rho\overleftrightarrow{\overline{u_i}}\;\overleftrightarrow{\overline{u_j}}$$

↓

$$\boxed{C_s^2 M_{ij} = L_{ij}}$$

⇒ **C_s dynamically obtained from the solution itself**

The dynamic procedure

The **dynamic** procedure can be applied **locally** :

- the constant depends on both **space** and **time**
- good for **complex** geometries,
- but **requires clipping (no warranty that the constant is positive)**

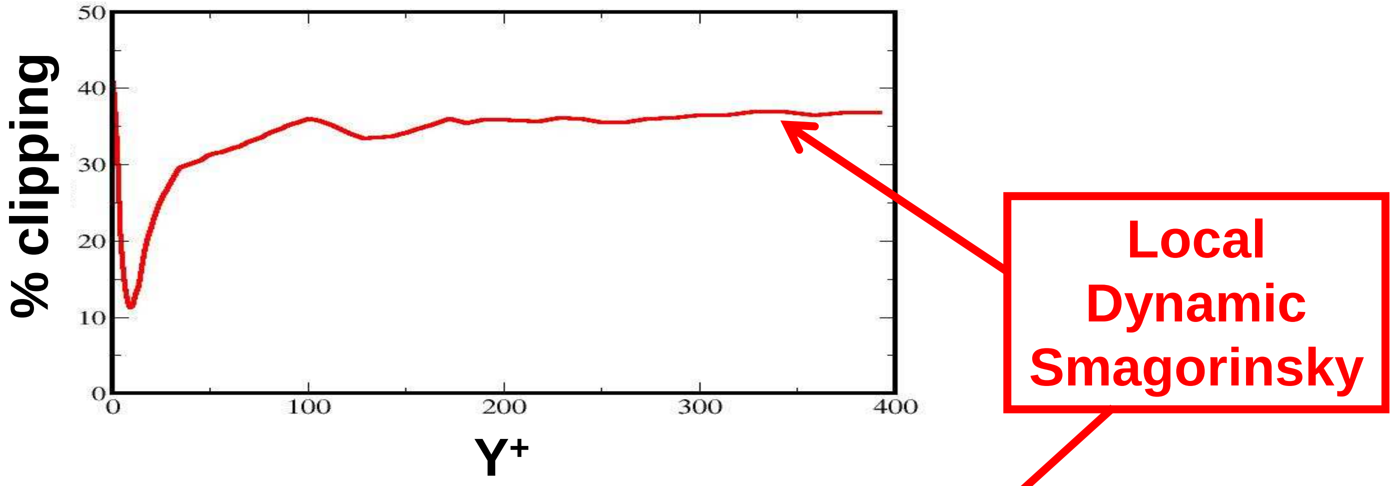
$$C^2 \equiv \frac{L_{ij} M_{ij}}{M_{ij} M_{ij}}$$

M_{ij} depends on the SGS model

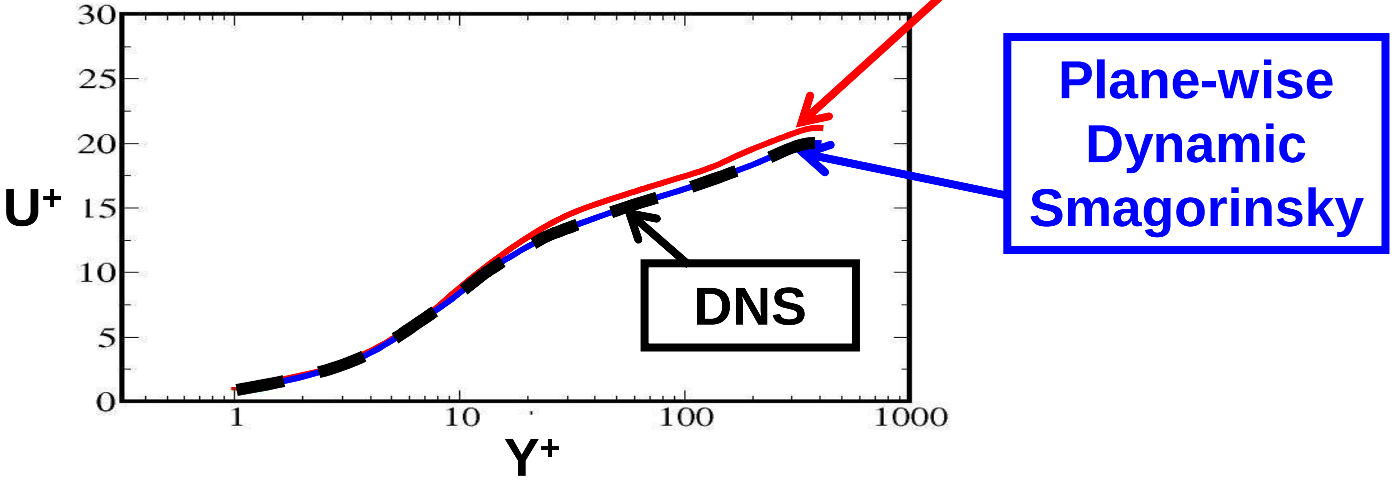
L_{ij} can be computed from the resolved scales

How often should we accept to clip ?

1. Example of a simple **turbulent** channel flow



2. **Not** very **satisfactory**, and may degrade the results



The global dynamic procedure

The **dynamic** procedure can also be applied **globally**:

- The constant depends only on **time**
- **no clipping** required,
- just as good as the **static** model it is based on
- Requires **an improved time scale estimate**

$$C^2 \equiv \frac{\langle L_{ij} M_{ij} \rangle}{\langle M_{ij} M_{ij} \rangle}$$

M_{ij} depends on the SGS model

L_{ij} can be computed from the resolved scales

Sub-grid scale model

1. Practically, **eddy-viscosity** models are often preferred
2. The gold standard today is the **Dynamic Smagorinsky** model

Dynamically Computed

From the grid or filter width

strain rate : $\sqrt{2S_{ij}S_{ij}}$

$$\mu_{sgs} = \rho C^2 \times \Delta^2 \times (\text{time scale})^{-1}$$

3. Looking for an improved model for the **time scale**

Description of the σ - model

- **Eddy-viscosity** based: $\mu_{sgs} = \rho (C\Delta)^2 \times (\text{time scale})^{-1}$
- Start to compute the **singular values** of the velocity **gradient tensor** (neither difficult nor expensive)
- $0 \leq \sigma_3 \leq \sigma_2 \leq \sigma_1$

Null for axi-symmetic flows

Null for isotropic

$$(\text{time scale})^{-1} = \frac{\sigma_3(\sigma_1 - \sigma_2)(\sigma_2 - \sigma_3)}{\sigma_1^2}$$

AND near-wall behavior is $O(y^3)$!!

Null for 2D or 2C flows

Null only if $g_{ij} = 0$

Sub-grid scale contributions

- Sub-grid scale stress tensor

$$\tau_{ij}^{sgs} = \bar{\rho}\,\tilde{u}_i\tilde{u}_j - \overline{\rho\, u_i u_j} \quad \rightarrow \quad \mu_{sgs} \text{ based model}$$

- Sub-grid scale mass flux of species k and heat flux

$$J_{k,j}^{sgs} = -\bar{\rho}\,\tilde{Y}_k\tilde{u}_j + \overline{\rho\, Y_k u_j} \quad \rightarrow \quad J_{k,j}^{SGS} = -\bar{\rho}\left(D_k^{SGS}\frac{W_k}{W}\frac{\partial \tilde{X}_k}{\partial x_j} - \tilde{Y}_k V_j^{c,SGS} \right)$$

$$q_j^{sgs} = -\bar{\rho}\,\tilde{E}\,\tilde{u}_j + \overline{\rho\, E\, u_j} \quad \rightarrow \quad q_j^{SGS} = -\lambda_{SGS}\frac{\partial \tilde{T}}{\partial x_j} + \sum_{k=1}^{N} J_{k,j}^{SGS}\,\tilde{h}_s^k$$

- In practice, constant SGS Schmidt and Prandtl numbers

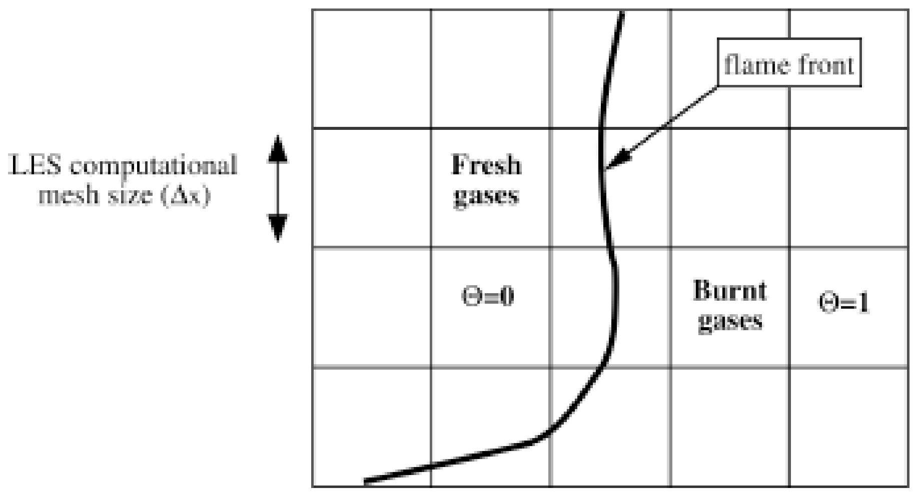
$$D_k^{sgs} = \frac{\mu_{sgs}}{\bar{\rho}\, Sc_k^{sgs}}; \quad Sc_k^{sgs} = 0.7 \qquad \lambda_{sgs} = \frac{\mu_{sgs} C_p}{\Pr_{sgs}}; \quad \Pr_{sgs} = 0.5$$

Sub-grid scale heat release

- The chemical source terms are highly non-linear (Arrhenius type of terms)

$$\dot{\omega}_F = -A_1 \rho^2 T^{\beta_1} Y_F Y_O \exp\left(-\frac{T_A}{T}\right)$$

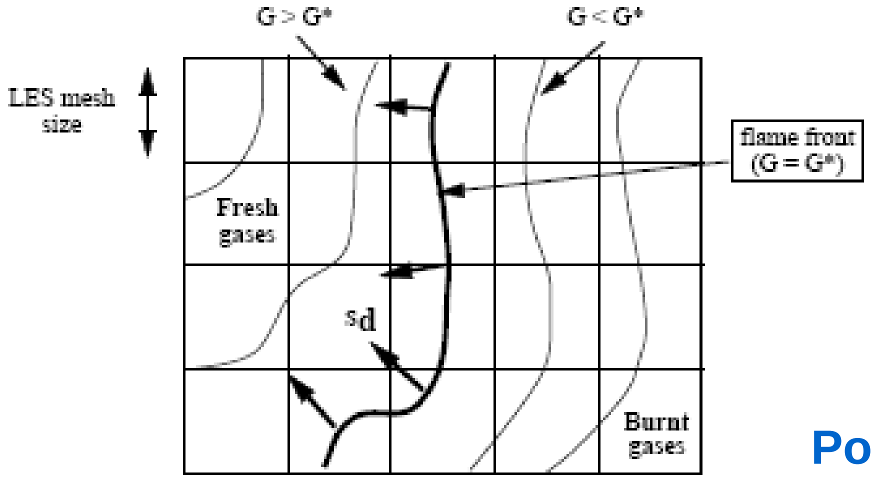
- The flame thickness is usually very small (0.1 - 1 mm), smaller than the typical grid size



Poinsot & Veynante, 2001

The G-equation approach

- The flame is identified as a given surface of a G field



Poinsot & Veynante, 2001

- The G-field is smooth and computed from

$$\frac{\partial \overline{\rho}\widetilde{G}}{\partial t} + \frac{\partial \overline{\rho}\widetilde{u}_i\widetilde{G}}{\partial x_i} = \rho_0 \overline{s}_T \left|\nabla \overline{G}\right|$$

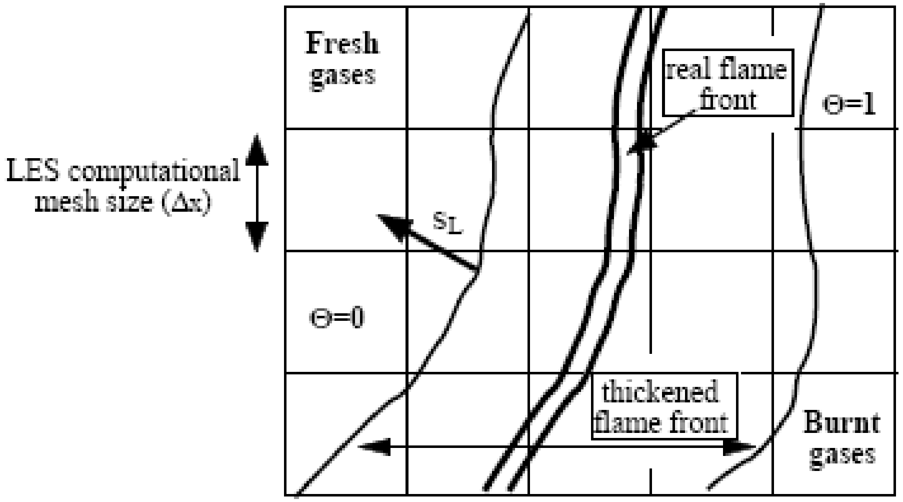
- “Universal “ turbulent flame speed model not available …

The thickened flame approach

- From laminar premixed flames theory:

$$S_L^o \propto \sqrt{a\,A} \quad \text{, and,} \quad \delta_L^0 \propto \frac{a}{S_L^0} \propto \sqrt{\frac{a}{A}},$$

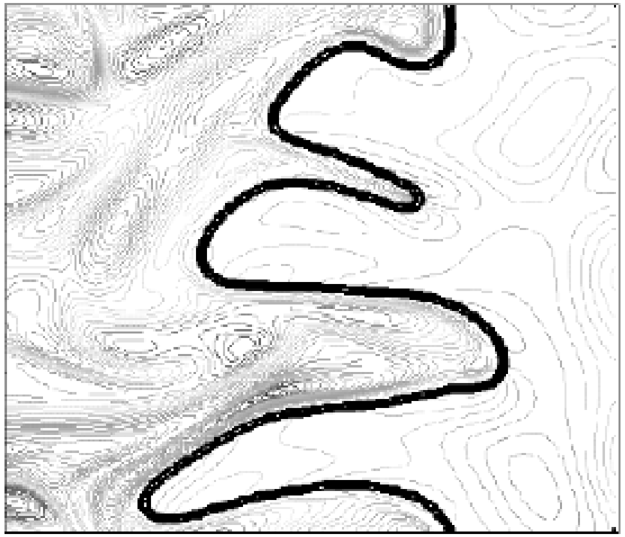
- Multiplying a and dividing A by the same thickening factor F



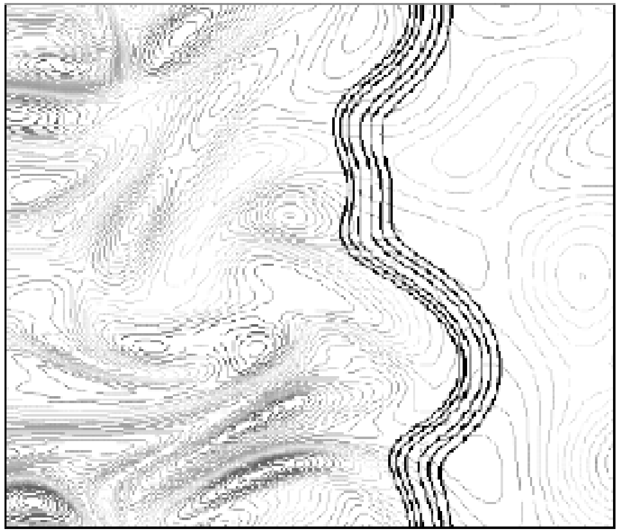
Poinsot & Veynante, 2001

The thickened flame approach

- The thickened flame propagates at the proper laminar speed but it is less wrinkled than the original flame:



Total reaction rate R_1



Total reaction rate R_2

- This leads to a decrease of the total consumption: $R_1 \rightarrow R_2 = \dfrac{R_1}{E} < R_1$

The thickened flame approach

- An efficiency function is used to represent the sub-grid scale wrinkling of the thickened flame

$$\text{Diffusivity}: \quad a \rightarrow Fa \rightarrow EFa$$

$$\text{Preexponential constant}: \quad A \rightarrow A/F \rightarrow EA/F$$

thickening SGS wrinckling

- This leads to a thickened flame (resolvable) with increased velocity (SGS wrinkling) with the proper total rate of consumption

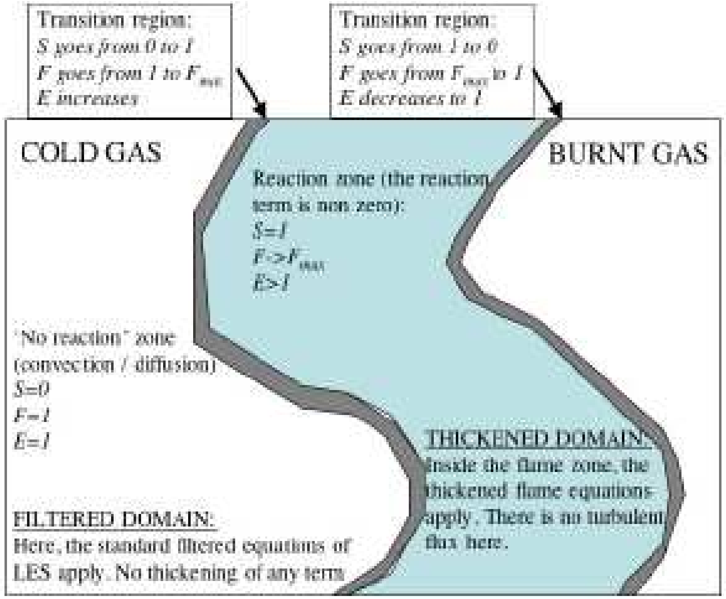
- The efficiency function is a function of characteristic velocity and length scales ratios

$$\frac{u_{sgs}}{S_L^0} \qquad \frac{\Delta}{F\delta_L^0}$$

The thickened flame approach

- **Advantages**
  1. finite rate chemistry (ignition / extinction)
  2. fully resolved flame front avoiding numerical problems
  3. easily implemented and validated
  4. degenerates towards DNS: does laminar flames

- **But the mixing process is not computed accurately outside the reaction zone**
  1. Extension required for diffusion or partially premixed flames
  2. Introduction of a sensor to detect the flame zone and switch the F and E terms off in the non-reacting zones

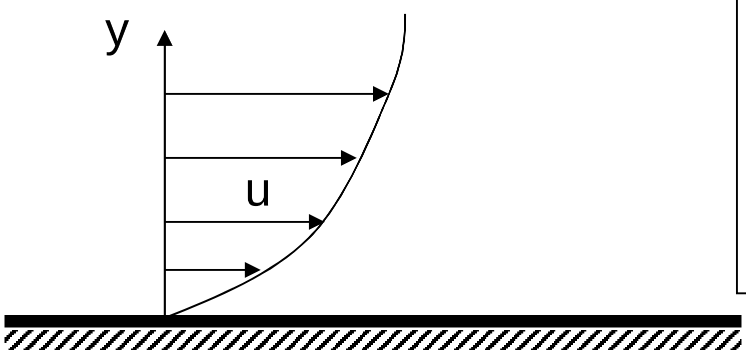


About solid walls

- In the near wall region, the total shear stress is constant. Thus the proper velocity and length scales are based on the wall shear stress τ_w:

$$u_\tau = \sqrt{\frac{\tau_w}{\rho}} \qquad\qquad l = \frac{\nu}{u_\tau}$$

- In the case of attached boundary layers, there is an inertial zone where the following universal velocity law is followed



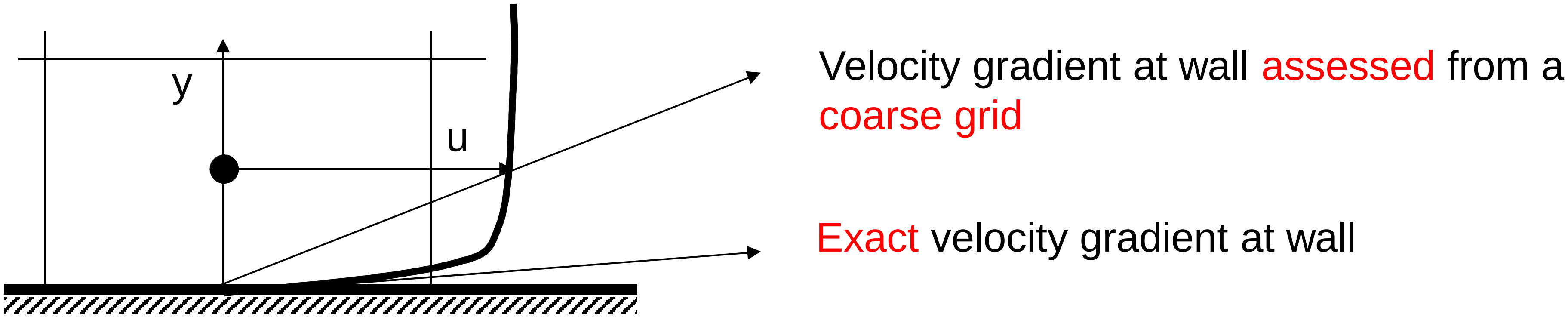
$$u^+ = \frac{1}{\kappa}\ln y^+ + C, \qquad u^+ = \frac{u}{u_\tau}, \qquad y^+ = \frac{y u_\tau}{\nu}$$

κ : Von Karman constant, $\kappa \approx 0.4$

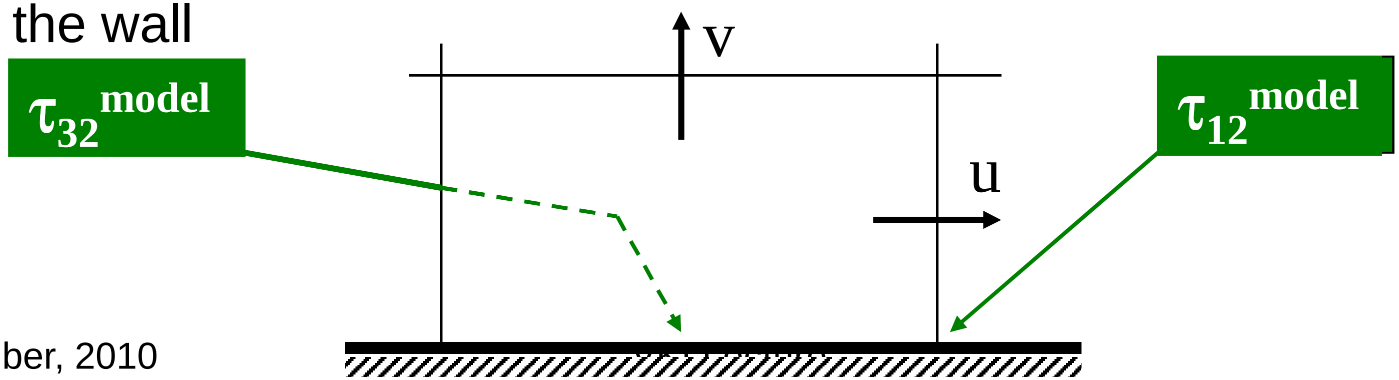
C : "Universal constant", $C \approx 5.2$

Wall modeling

- A specific wall treatment is required to avoid huge mesh refinement or large errors,



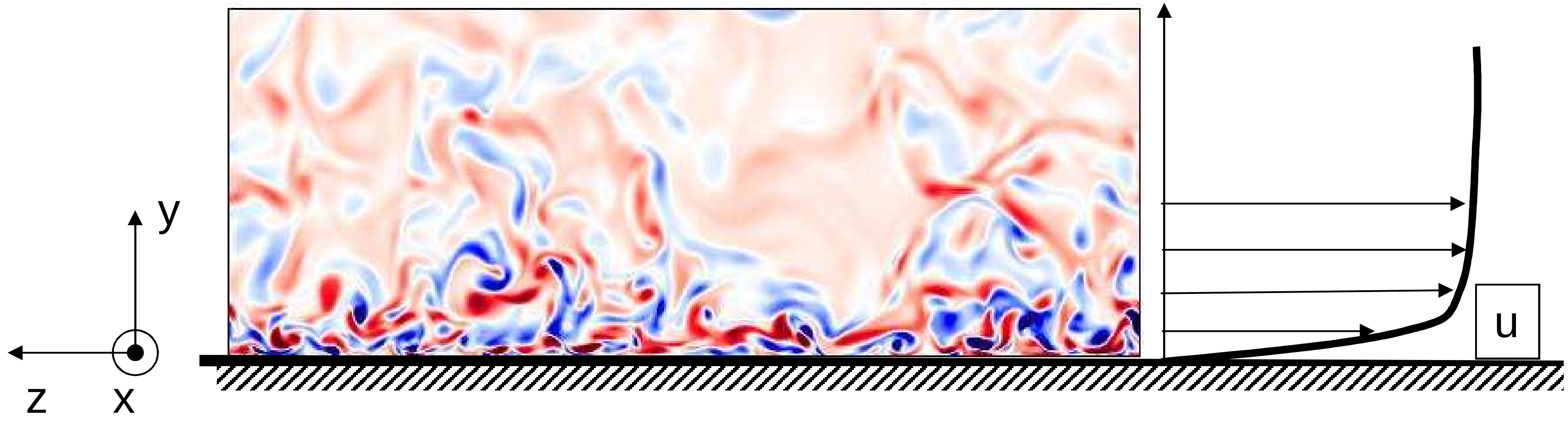
- Use a coarse grid and the log law to impose the proper fluxes at the wall



About solid walls

- Close to solid walls, the largest scales are small …

Boundary layer along the x-direction: Vorticity ω_x



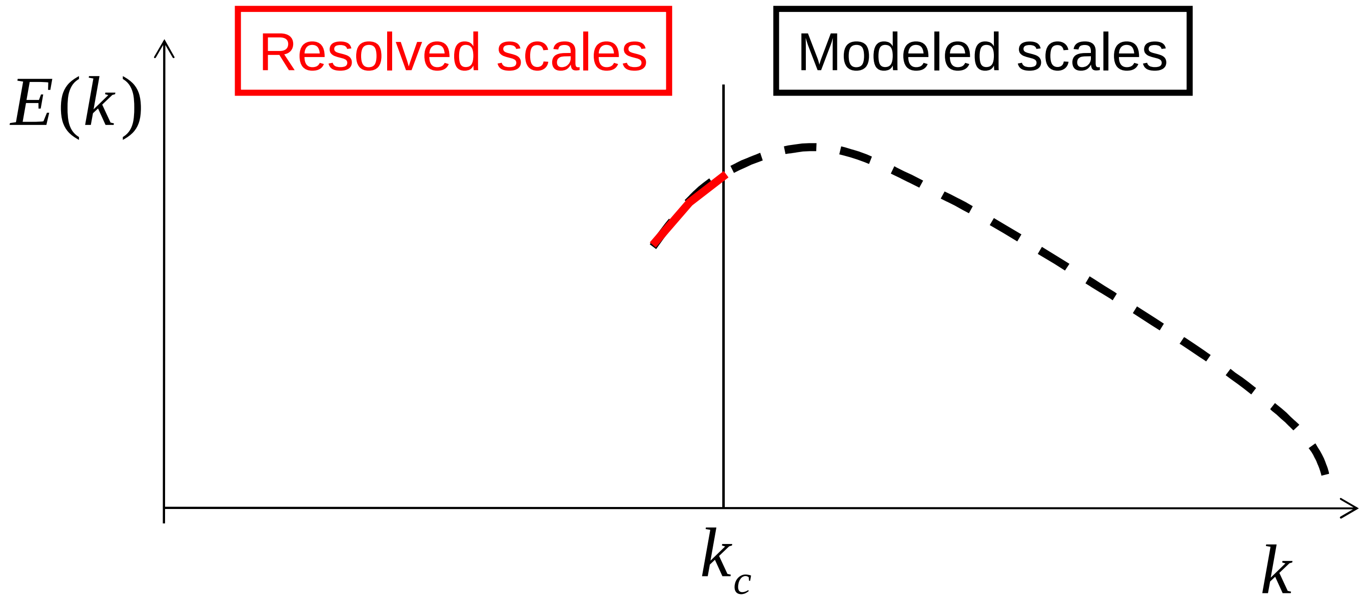
- steep velocity profile, $L_t \sim \kappa y$

- Resolution requirement: $\Delta y^+ = O(1)$, Δz^+ **and** $\Delta x^+ = O(10)$!!

- Number of grid points: $O(R_\tau^2)$ for wall resolved LES

Wall modeling in LES

- Not even the **most energetic** scales are resolved when the first off-wall point is in the log layer
- No reliable model available yet

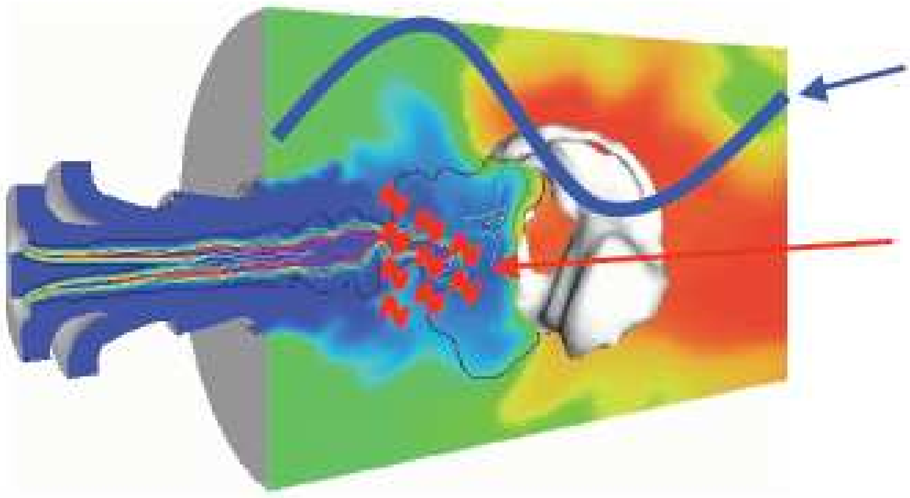


OUTLINE

1. Computing the whole flow
2. Computing the fluctuations
3. Boundary conditions
4. Analysis of an annular combustor

Considering only perturbations

Approach: solve acoustic field using finite volume method



resolve only large scales (acoustic modes)

model small scales (combustion)

⇒ **Compared to LES:**

- simplified system of equations, coarser grid

- requires less computational time

Linearized Euler Equations

$$\frac{D\rho}{Dt} = -\rho\nabla\cdot\mathbf{u}$$

$$\rho\frac{D\mathbf{u}}{Dt} = -\nabla p$$

$$\frac{Ds}{Dt} = \frac{rq}{p}$$

- **assume homogeneous mixture**
- **neglect viscosity**

- **decompose each variable into its mean and fluctuation**

$$f(\mathbf{x},t) = f_0(\mathbf{x}) + f_1(\mathbf{x},t)$$

- **assume small amplitude fluctuations**

$$\frac{f_1}{f_0} \equiv \varepsilon << 1; \quad f = \rho, p, T, s = C_v \ln\left(\frac{p}{\rho^\gamma}\right)$$

$$\frac{\|\mathbf{u}_1\|}{c_0} \equiv \varepsilon << 1; \quad c_0 = \sqrt{\frac{\gamma p_0}{\rho_0}}$$

Linearized Euler Equations

$$\frac{\partial \rho_1}{\partial t} + \mathbf{u}_0 \nabla \rho_1 + \mathbf{u}_1 \nabla \rho_0 + \rho_0 \nabla \cdot \mathbf{u}_1 + \rho_1 \nabla \cdot \mathbf{u}_0 = 0$$

$$\rho_0 \frac{\partial \mathbf{u}_1}{\partial t} + \rho_0 \mathbf{u}_0 \cdot \nabla \mathbf{u}_1 + \rho_0 \mathbf{u}_1 \cdot \nabla \mathbf{u}_0 + \rho_1 \mathbf{u}_0 \cdot \nabla \mathbf{u}_0 + \nabla p_1 = 0$$

$$\frac{\partial s_1}{\partial t} + \mathbf{u}_0 \nabla s_1 + \mathbf{u}_1 \nabla s_0 = \frac{r q_1}{p_0} - \frac{r q_0 p_1}{p_0^2}$$

- the unknown are the small amplitude fluctuations,
- the mean flow quantities must be provided
- requires a model for the heat release fluctuation q_1
- contain all what is needed, and more …: acoustics + vorticity + entropy

Zero Mach number assumption

- No mean flow or “Zero-Mach number” assumption

$$f(\mathbf{x},t) = f_0(\mathbf{x}) + f_1(\mathbf{x},t); \quad \frac{f_1}{f_0} \equiv \varepsilon << 1; \quad f = \rho, p, T, s = C_v \ln\left(\frac{p}{\rho^\gamma}\right)$$

$$\mathbf{u}(\mathbf{x},t) = \mathbf{u}_0(\mathbf{x}) + \mathbf{u}_1(\mathbf{x},t); \quad \frac{\|\mathbf{u}_1\|}{c_0} \equiv \varepsilon << 1; \quad c_0 = \sqrt{\frac{\gamma p_0}{\rho_0}}$$

Equation	Constraint
mass	$M \ll 1$ and $M \ll L_f/L_a$
momentum	$M \ll L_f/L_a$; $M \ll 1$ and $M \ll \sqrt{L_f/L_a}$
entropy	$M \ll 1$

L_a : acoustic wavelength $\quad L_f$: flame thickness

- Probably well justified below 0.01

Linear equations

$$\text{Mass}: \quad \frac{\partial \rho_1}{\partial t} + \rho_0 div(\mathbf{u}_1) + \mathbf{u}_1 \cdot \nabla \rho_0 = 0 \qquad \text{Momentum}: \quad \rho_0 \frac{\partial \mathbf{u}_1}{\partial t} = -\nabla p_1$$

$$\text{Energy}: \quad \rho_0 C_v \left[\frac{\partial T_1}{\partial t} + \mathbf{u}_1 \cdot \nabla T_0 \right] = -p_0 \nabla \cdot \mathbf{u}_1 + q_1 \qquad \text{State}: \quad \frac{p_1}{p_0} = \frac{\rho_1}{\rho_0} + \frac{T_1}{T_0}$$

- The unknowns are the fluctuating quantities $\rho_1, \mathbf{u}_1, T_1, p_1$

- The mean density, temperature, … fields must be provided

- A model for the unsteady HR q_1 is required to close the system

Flame Transfer Functions

- Relate the global HR $Q_1(t) = \int_\Omega q_1(\mathbf{x}, t) d\Omega$ to upstream velocity fluctuations

- General form in the frequency space

$$\boxed{\hat{Q}(\omega) = F(\omega) \times \hat{u}(\mathbf{x}_{ref}, \omega)}$$

- n-τ model (Crocco, 1956), low-pass filter/saturation (Dowling, 1997), laminar conic or V-flames (Schuller et al, 2003), entropy waves (Dowling, 1995; Polifke, 2001), …

- Justified for acoustically compact flames

Local FTF model

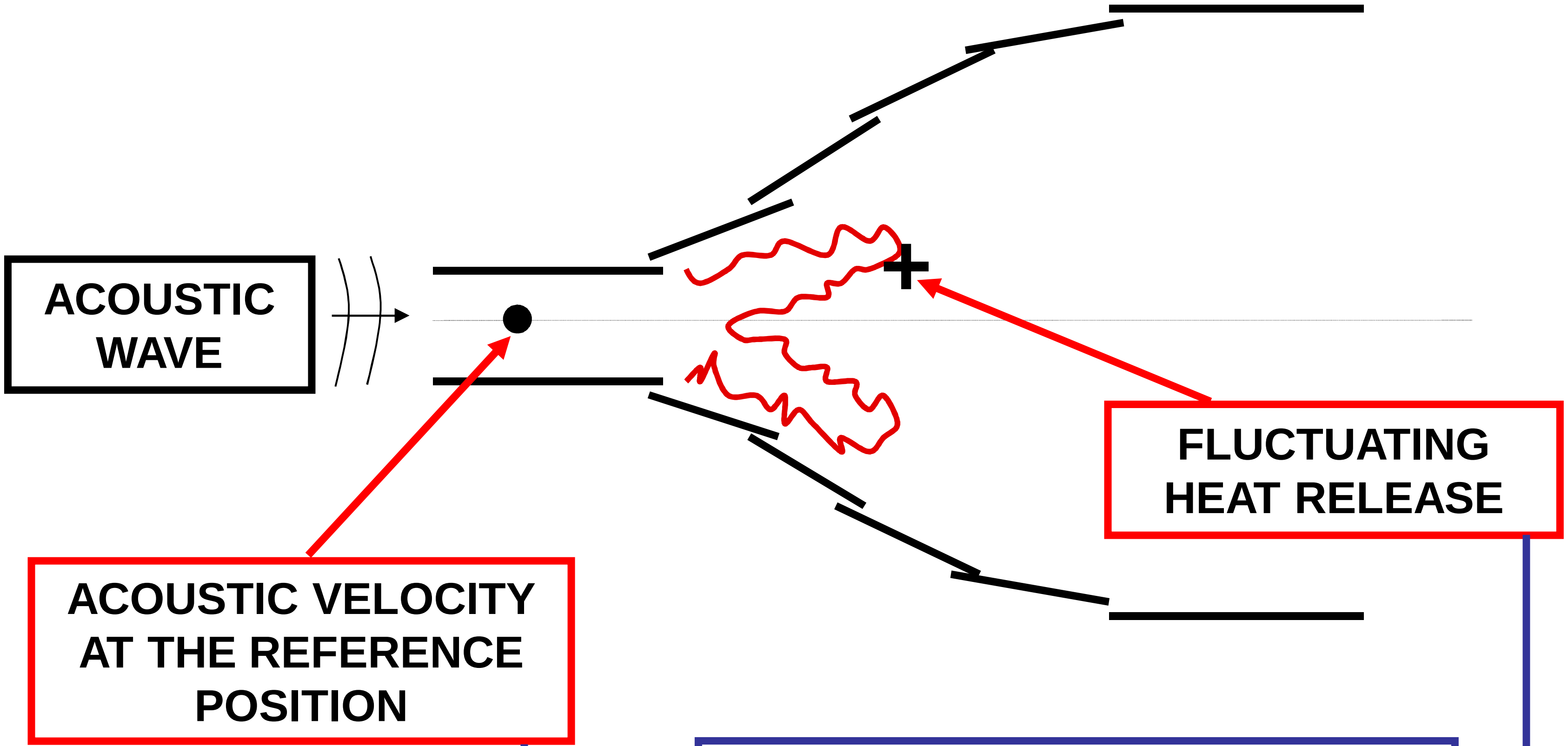
- Flame not necessarily compact
- Local FTF model

$$\frac{\hat{q}(\mathbf{x},\omega)}{q_{\text{mean}}} = n_l(\mathbf{x},\omega) \times e^{j\omega\tau_l(\mathbf{x},\omega)} \times \frac{\hat{\mathbf{u}}(\mathbf{x}_{\text{ref}},\omega)\cdot\mathbf{n}_{\text{ref}}}{U_{\text{bulk}}}$$

n_l, τ_l : **Two scalar fields**

- The scalar fields must be defined in order to match the actual flame response
- LES is the most appropriate tool assessing these fields

Local FTF model



$$n_l(\mathbf{x},\omega)\times e^{j\omega\tau_l(\mathbf{x},\omega)} \propto \frac{\hat{\omega}_T(\mathbf{x},\omega)}{\hat{\mathbf{u}}(\mathbf{x}_{\mathrm{ref}},\omega)\cdot\mathbf{n}_{\mathrm{ref}}}$$

Giauque et al., AIAA paper 2008-2943

Back to the linear equations

- Let us suppose that we have a "reasonable" model for the flame response

- The set of linear equations still needs to be solved

$$\text{Mass}: \quad \frac{\partial \rho_1}{\partial t} + \rho_0 div(\mathbf{u}_1) + \mathbf{u}_1 \cdot \nabla \rho_0 = 0$$

$$\text{Momentum}: \quad \rho_0 \frac{\partial \mathbf{u}_1}{\partial t} = -\nabla p_1$$

$$\text{State}: \quad \frac{p_1}{p_0} = \frac{\rho_1}{\rho_0} + \frac{T_1}{T_0}$$

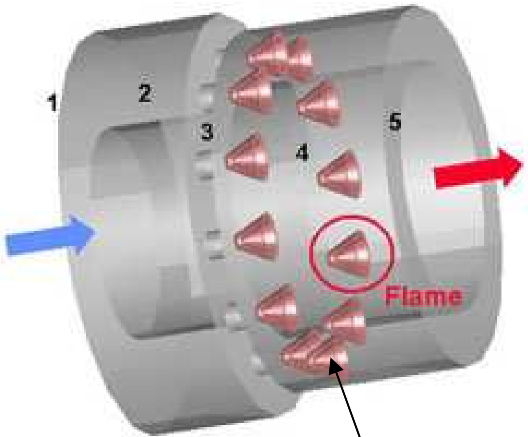
$$\text{Energy}: \quad \rho_0 C_v \left[\frac{\partial T_1}{\partial t} + \mathbf{u}_1 \cdot \nabla T_0 \right] = -p_0 \nabla \cdot \mathbf{u}_1 + q_1$$

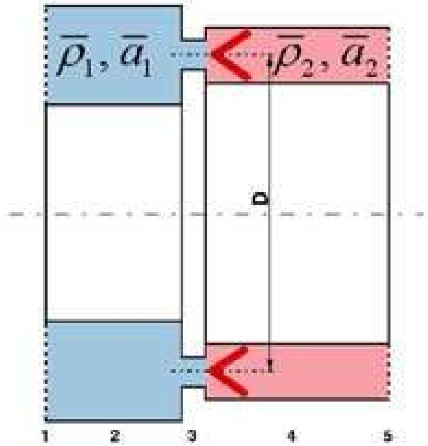
Time domain integration

- Use a finite element mesh of the geometry
- Prescribe boundary conditions
- Initialize with random fields
- Compute its evolution over time …

- This is the usual approach in LES/CFD !

A simple annular combustor (TUM)

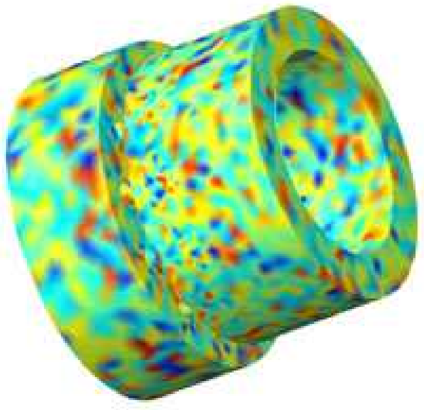




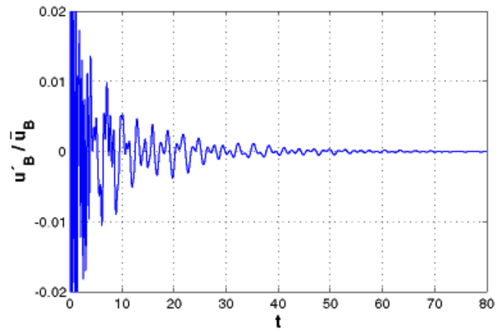
Unsteady flame model with characteristic time delay τ

Pankiewitz and Sattelmayer, J. Eng. Gas Turbines and Power, 2003

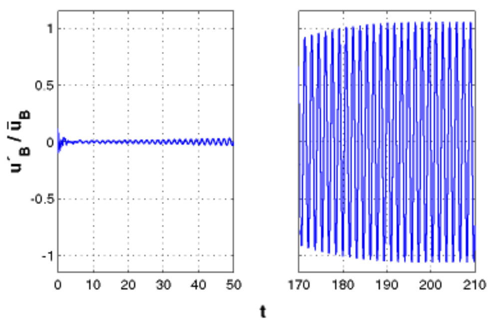
Example of time domain integration



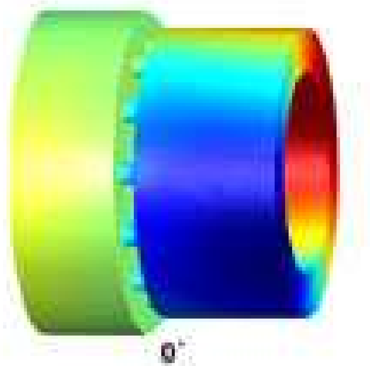
TIME DELAY LEADS TO STABLE CONDITIONS



TIME DELAY LEADS TO UNSTABLE CONDITIONS



ORGANIZED ACOUSTIC FIELD



Pankiewitz and Sattelmayer, J. Eng. Gas Turbines and Power, 2003

The Helmholtz equation

- Since 'periodic' fluctuations are expected, let's work in the frequency space

$$p_1(\mathbf{x},t) = \Re\left(\hat{p}(\mathbf{x})e^{-j\omega t}\right) \qquad \mathbf{u}_1(\mathbf{x},t) = \Re\left(\hat{\mathbf{u}}(\mathbf{x})e^{-j\omega t}\right)$$

$$q_1(\mathbf{x},t) = \Re\left(\hat{q}(\mathbf{x})e^{-j\omega t}\right)$$

- From the set of linear equations for ρ_1, $\mathbf{u}_1$, p_1, T_1 , the following wave equation can be derived

$$\boxed{\gamma p_0 \nabla \cdot \left(\frac{1}{\rho_0}\vec{\nabla}\hat{p}\right) + \omega^2 \hat{p} = j\omega(\gamma - 1)\hat{q}}$$

3D acoustic codes

- Let us first consider the simple ‘steady flame’ case

$$\boxed{\gamma p_0 \nabla \cdot \left(\frac{1}{\rho_0} \vec{\nabla} \hat{p} \right) + \omega^2 \hat{p} = 0}$$

- With simple boundary conditions

$$\hat{p} = 0 \quad \text{or} \quad \rho_0 \omega \, \hat{\mathbf{u}} \cdot \mathbf{n} = \nabla \hat{p} \cdot \mathbf{n} = 0$$

- Use the Finite Element framework to handle complex geometries

Discrete problem

- If m is the number of nodes in the mesh, the unknown is now

$$\mathbf{P} = \left[\hat{p}_1, \hat{p}_2, \ldots, \hat{p}_m\right]^T$$

- Applying the FE method, one obtains

Linear Eigenvalue Problem of size N

$$\mathbf{AP} + \omega^2 \mathbf{P} = 0 \qquad \mathbf{A}: \text{square matrix}$$

$$\text{discrete version of}: \gamma p_0 \nabla \cdot \left(\frac{1}{\rho_0} \nabla \right)$$

Solving the eigenvalue problem

- The QR algorithm is the method of choice for small/medium scale problems

  Shur decomposition: **AQ=QT,** Q unitary, T upper triangular

- Krylov-based algorithms are more appropriate when only a few modes needs to be computed

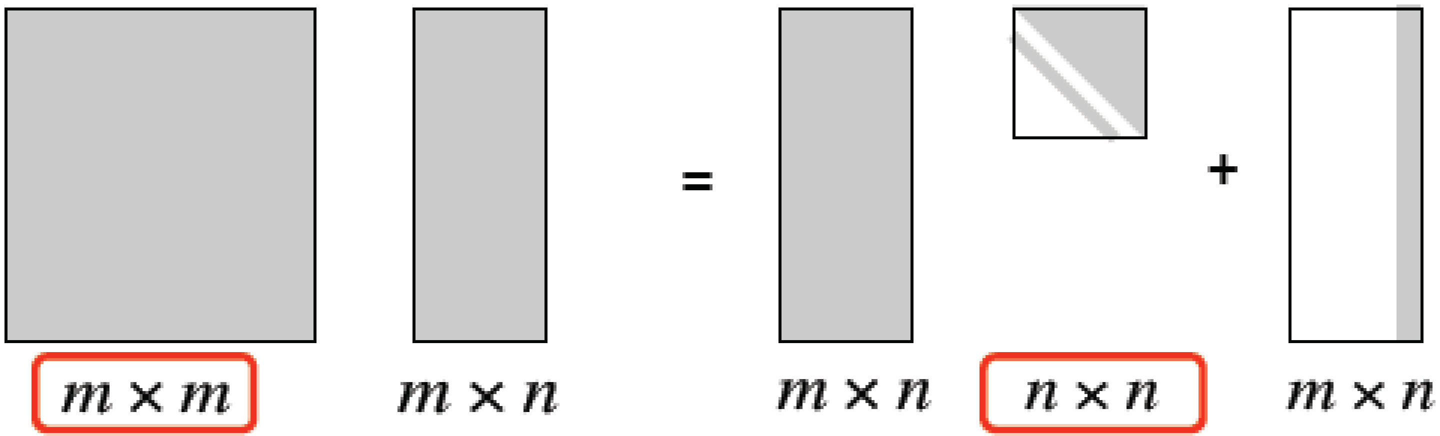
  Partial Shur decomposition: $\mathbf{AQ_n=Q_nH_n+E_n}$ with $n << m$

- A possible choice: the Arnoldi method implemented in the P-ARPACK library (Lehoucq et al., 1996)

Solving the eigenvalue problem

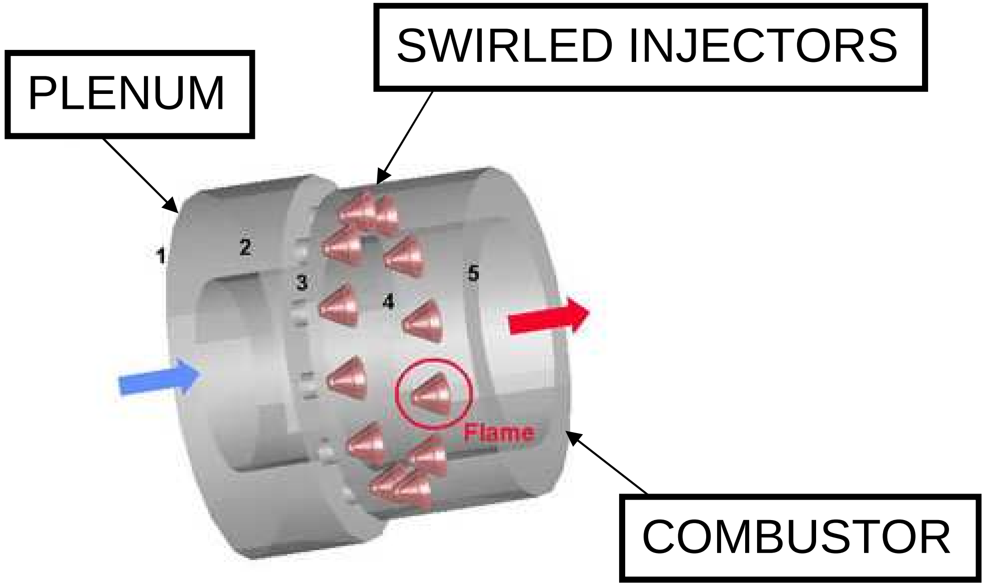
$$A\hat{v}(x) = i\omega\hat{v}(x)$$

$$A \quad Q_n \quad = \quad Q_n \quad H_n \quad + \quad h_{n+1,n} q_{n+1} e_n^T$$



eigenvalues and ~vectors of A ⟹ eigenvalues of H: **approximation of eigenvalues of A**

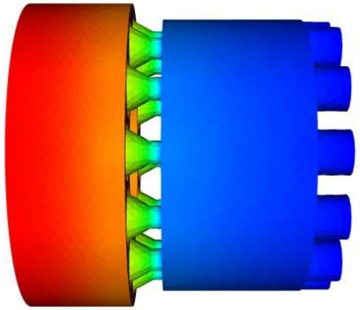
Computing the TUM annular combustor



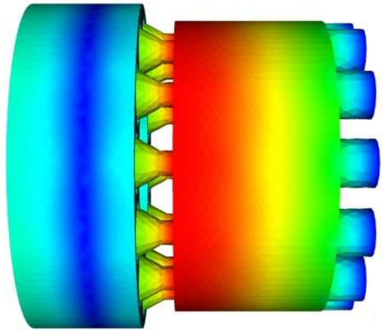
- **Experimentally, the first 2 modes are :**
  - **150 Hz (1L)**
  - **300 Hz (1C plenum)**

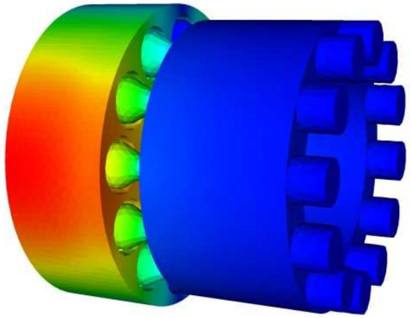
- FE mesh of the plenum + 12 injectors + swirlers + combustor + 12 nozzles
- Mean temperature field prescribed from experimental observations

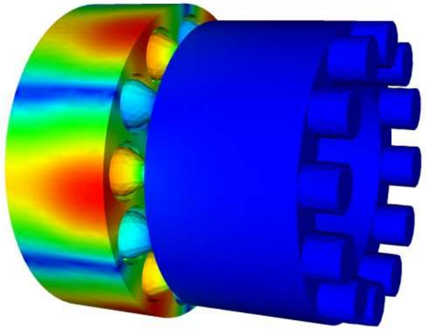
TUM combustor: first seven modes

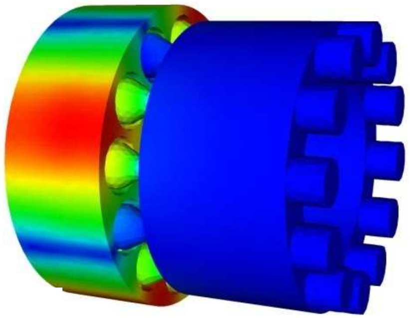


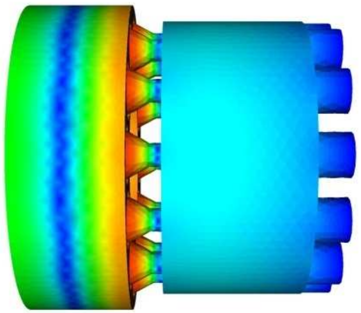
f (Hz)	type
143	1L
286	1C plenum
503	2C plenum
594	2L
713	3C plenum
754	1C combustor
769	3L

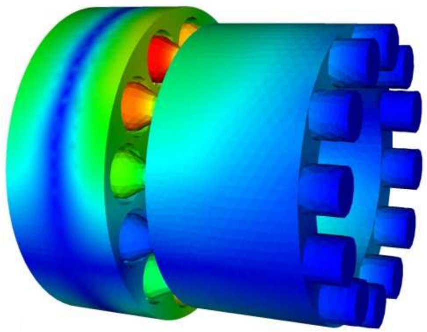




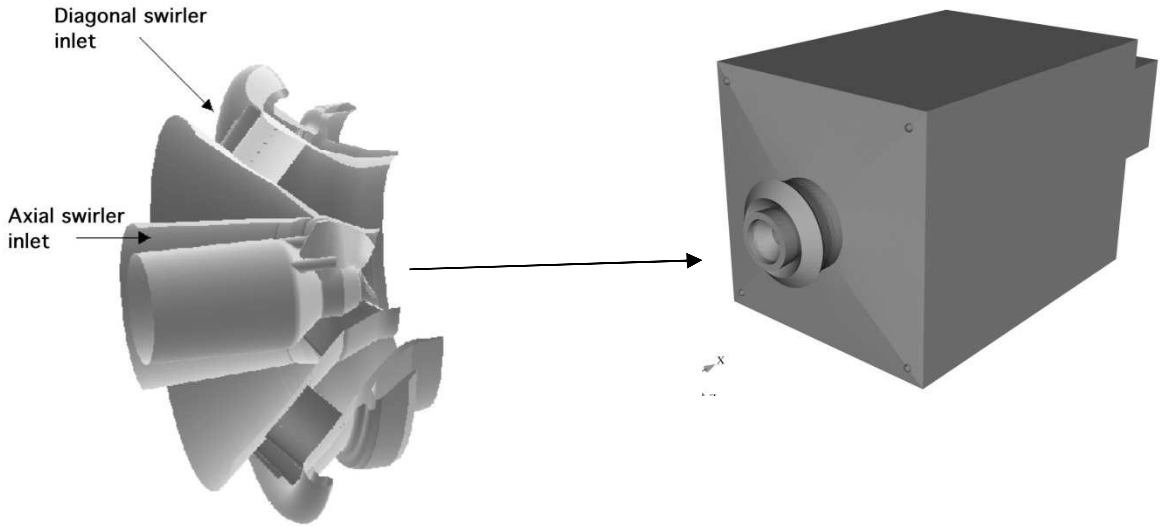








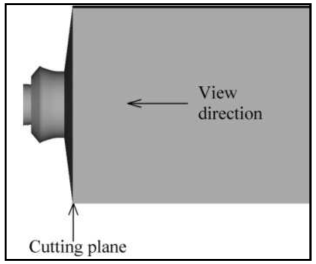
An industrial gas turbine burner

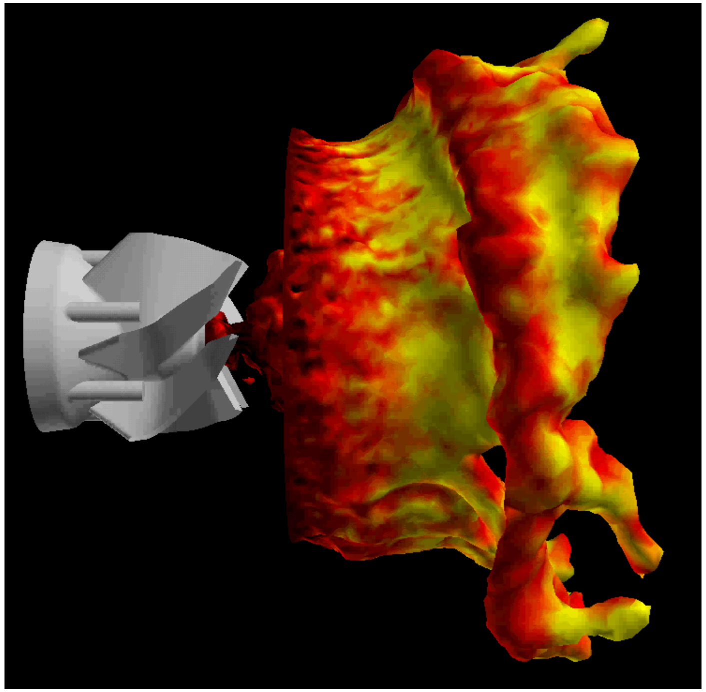


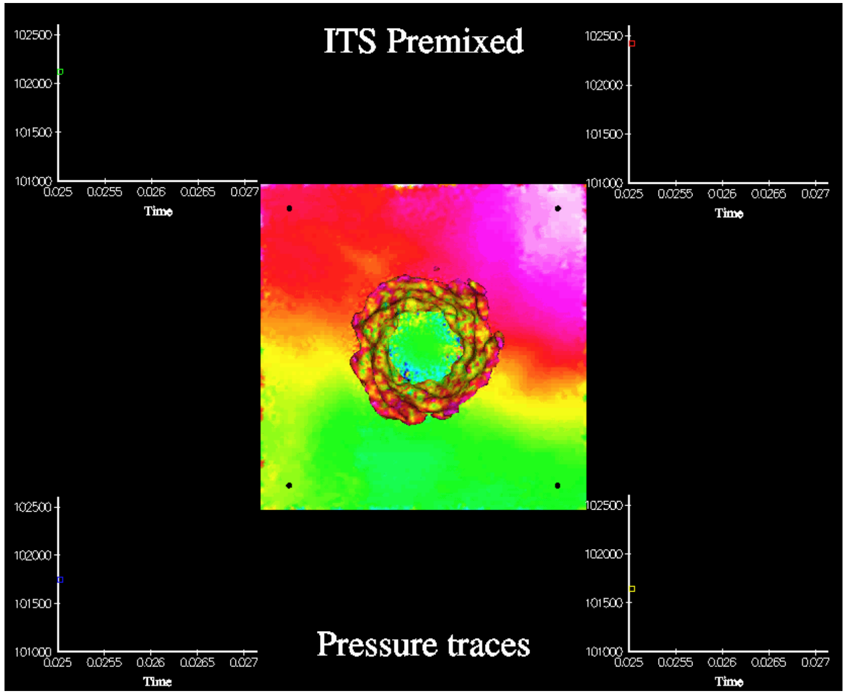
- Industrial burner (Siemens) mounted on a square cross section combustion chamber
- Studied experimentally (Schildmacher et al., 2000), by LES (Selle et al., 2004), and acoustically (Selle et al., 2005)

Instability in the LES

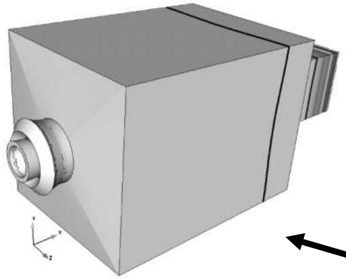
- From the LES, an instability develops at 1198 Hz

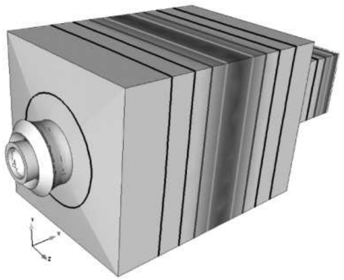


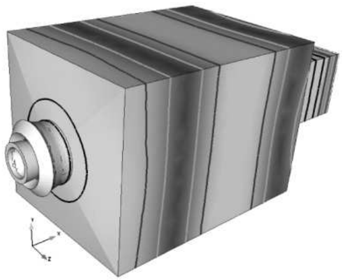




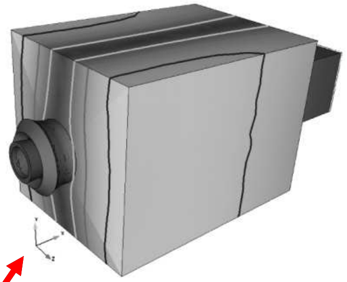
Acoustic modes

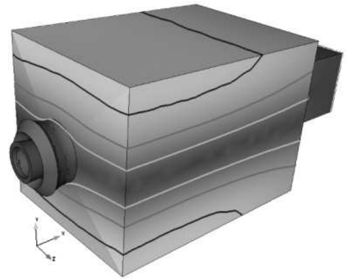


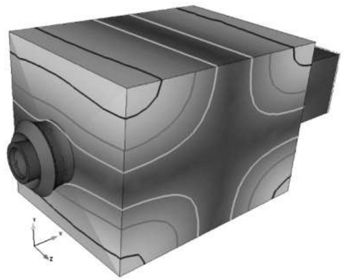


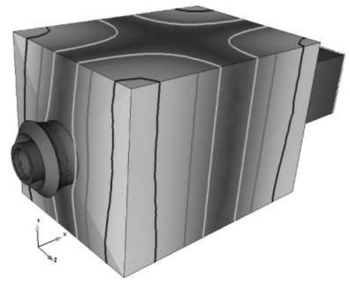


f (Hz)	type
159	1L
750	2L
1192	1L1T
1192	1L1T'
1350	3L
1448	2L1T
1448	2L1T'





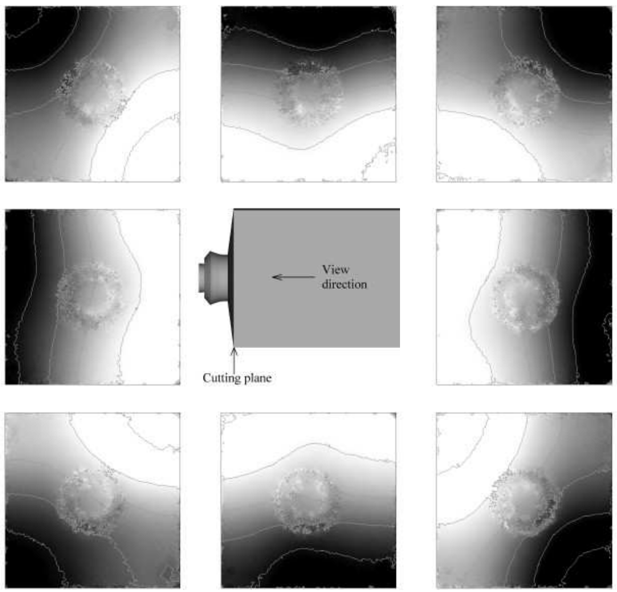




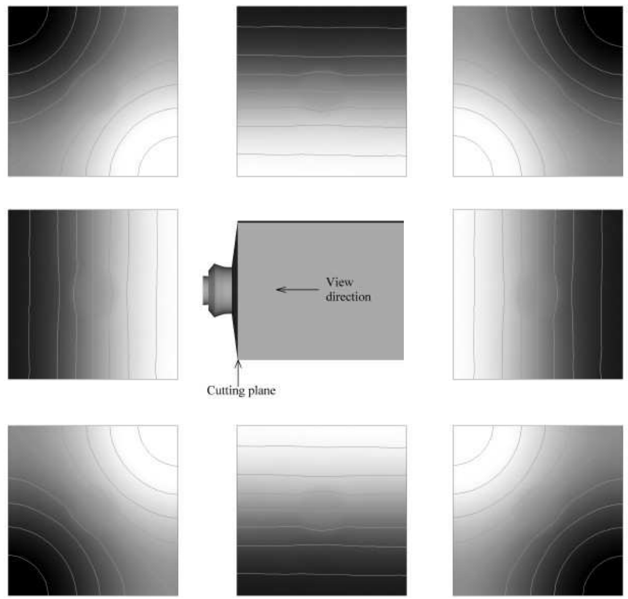
Turning mode

- The turning mode observed in LES can be recovered by adding the two 1192 Hz modes with a 90° phase shift

PRESSURE FLUCTUATIONS FOR 8 PHASES



LES SOLVER: 1198 Hz



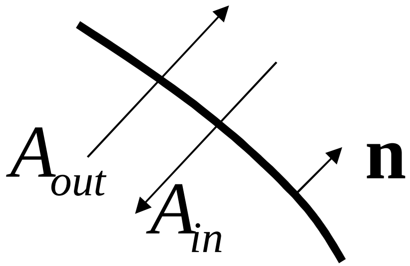
ACOUSTIC SOLVER: 1192 Hz

Selle et al., 2005

Realistic boundary conditions

- Complex, reduced boundary impedance

$$Z = \frac{\hat{p}}{\rho c \hat{\mathbf{u}} \cdot \mathbf{n}}$$



$$e.g.: \quad Z = 0 \Leftrightarrow \hat{p} = 0$$
$$Z = \infty \Leftrightarrow \hat{\mathbf{u}} \cdot \mathbf{n} = 0$$

- Reflection coefficient

$$R = \frac{A_{in}}{A_{out}} = \frac{Z-1}{Z+1}$$

- Using the momentum equation, the most general BC is

$$\boxed{\nabla \hat{p} \cdot \mathbf{n} - \frac{j\omega}{cZ} \hat{p} = 0}$$

Discrete problem with realistic BCs

- In general, the reduced impedance depends on ω and the discrete EV problem becomes non-linear:

$$\mathbf{AP} + \underbrace{\text{Boundary Terms}}_{\nabla \hat{p} \cdot \mathbf{n} = \frac{j\omega}{cZ(\omega)} \hat{p}} + \omega^2 \mathbf{P} = 0$$

- Assuming $\frac{1}{Z(\omega)} = \frac{1}{Z_0} + C_1 \omega + \frac{C_2}{\omega}$; Z_0, C_1, C_2 parameters

$$\mathbf{AP} + \omega \mathbf{BP} + \omega^2 \mathbf{CP} = 0$$

$$\mathbf{A}, \mathbf{B}, \mathbf{C} : \text{square matrices}$$

Quadratic Eigenvalue Problem of size N

From quadratic to linear EVP

- Given a quadratic EVP of size *N:* $\mathbf{AP}+\omega\mathbf{BP}+\omega^2\mathbf{CP}=0$

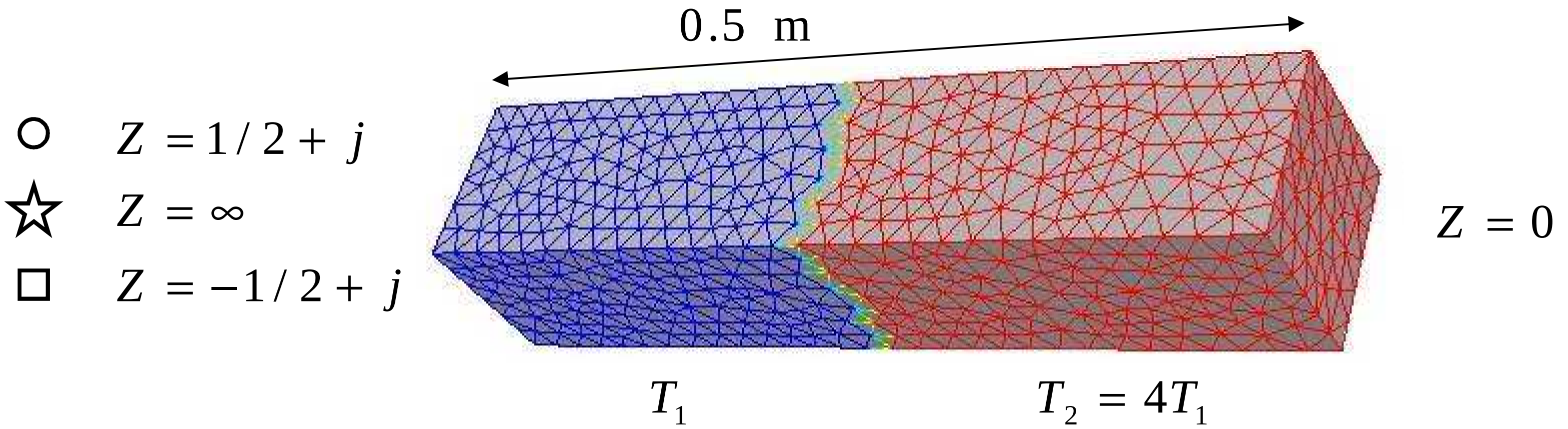
- Add the variable: $\mathbf{R}=\omega\,\mathbf{P}$

- Rewrite:

$$\left.\begin{aligned}-\mathbf{IR}+\omega\,\mathbf{IP}=0\\ \mathbf{AP}+\mathbf{BR}+\omega\mathbf{CR}=0\end{aligned}\right\}\Rightarrow\begin{bmatrix}\mathbf{0} & -\mathbf{I}\\ \mathbf{A} & \mathbf{B}\end{bmatrix}\begin{bmatrix}\mathbf{P}\\ \mathbf{R}\end{bmatrix}+\omega\begin{bmatrix}\mathbf{I} & \mathbf{0}\\ \mathbf{0} & \mathbf{C}\end{bmatrix}\begin{bmatrix}\mathbf{P}\\ \mathbf{R}\end{bmatrix}=0$$

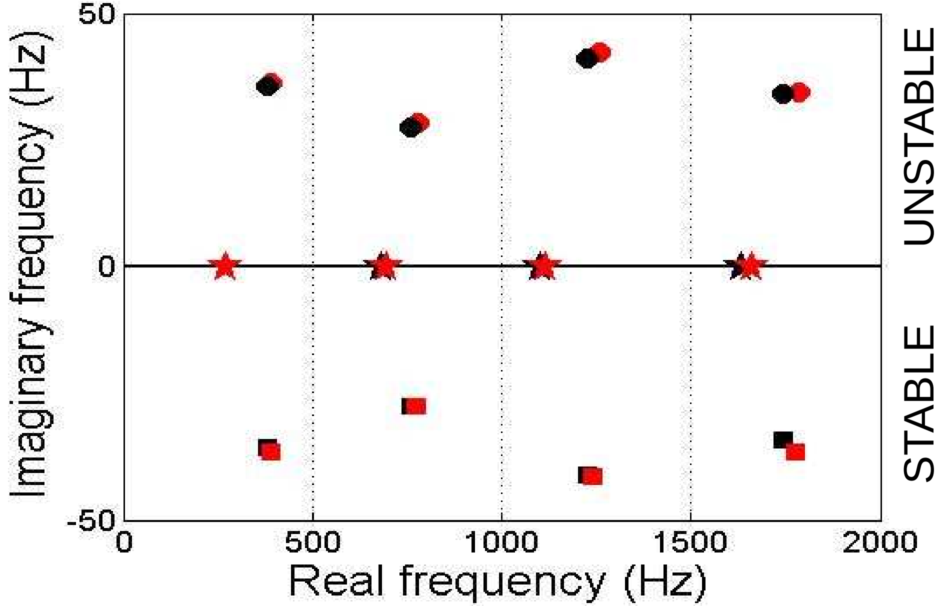
- Obtain an equivalent Linear EVP of size *2N* and use your favorite method !!

Academic validation



ACOUSTIC SOLVER

EXACT

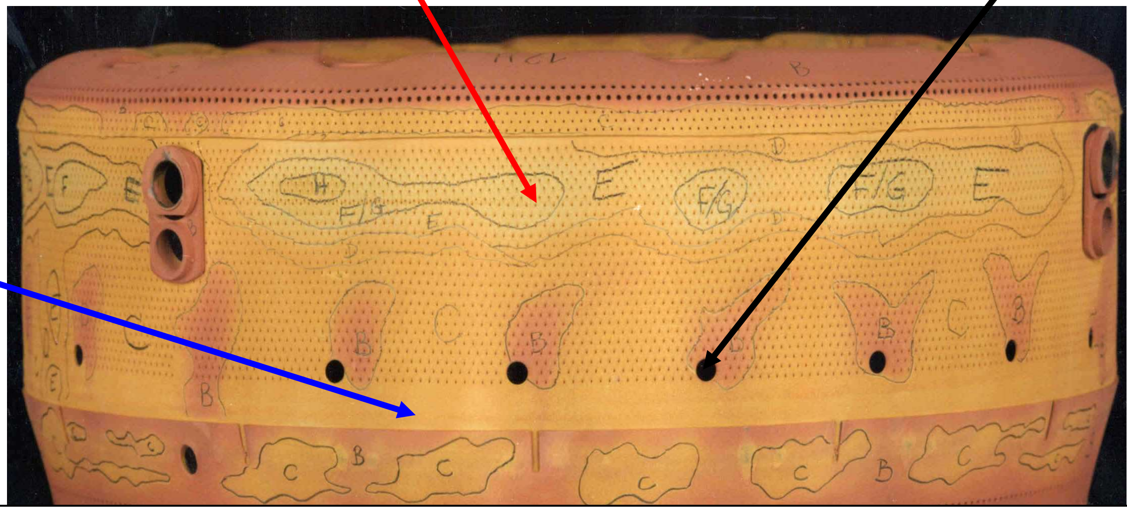


Multiperforated liners

Multiperforated plate
(????)

Dilution hole
(resolved)

Solid plate
(Neumann)



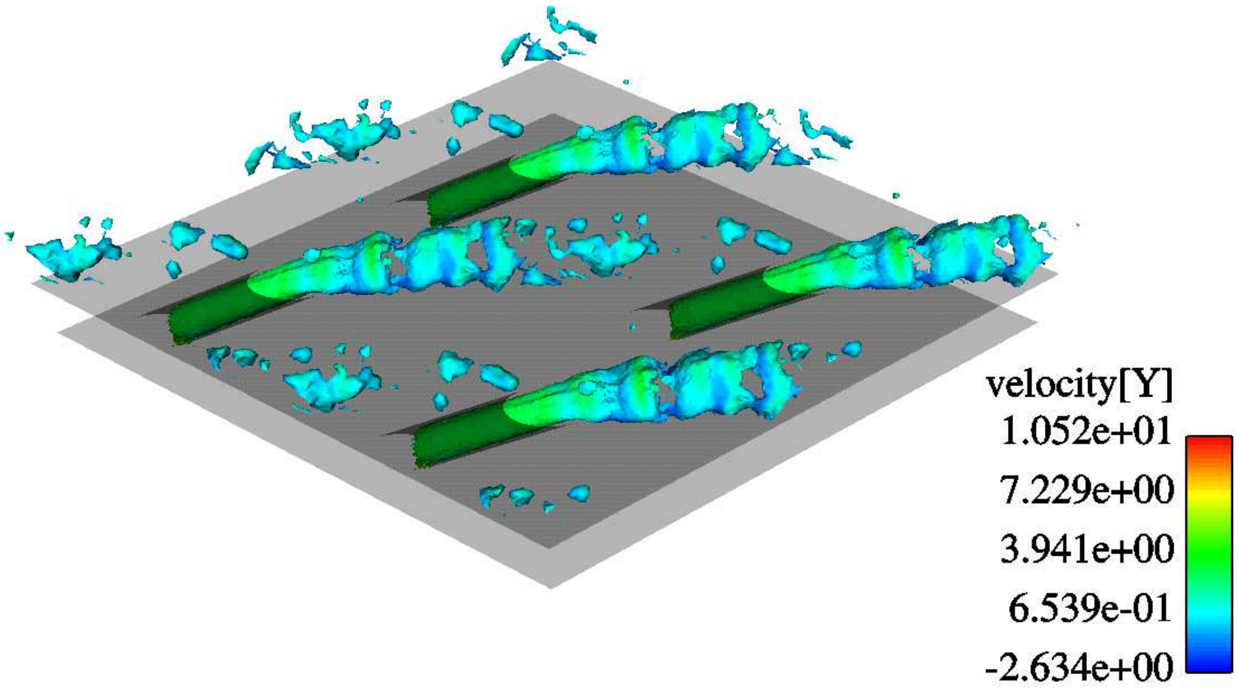
Example of a TURBOMECA burner

Multiperforated liners

- Designed for cooling purpose …

Burnt gas (combustion chamber)

Cold gas (casing)

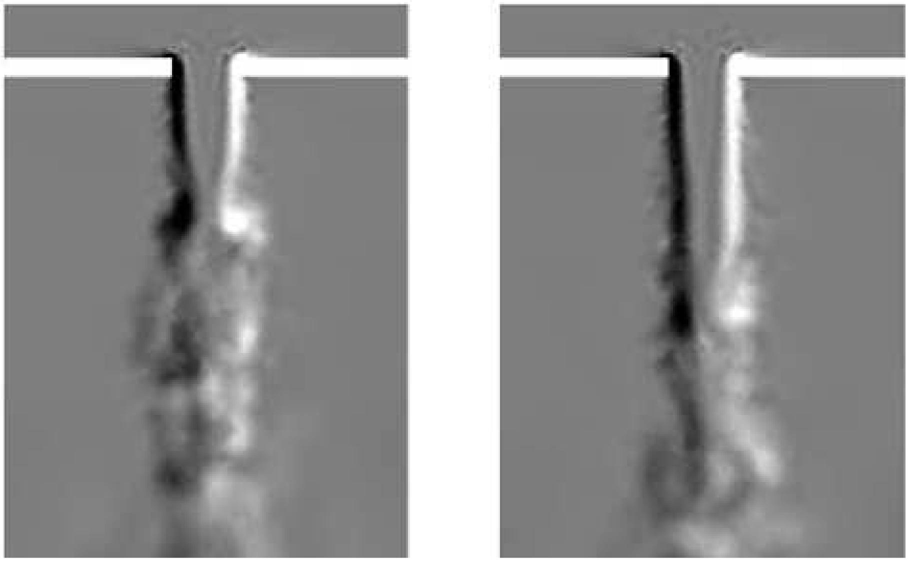


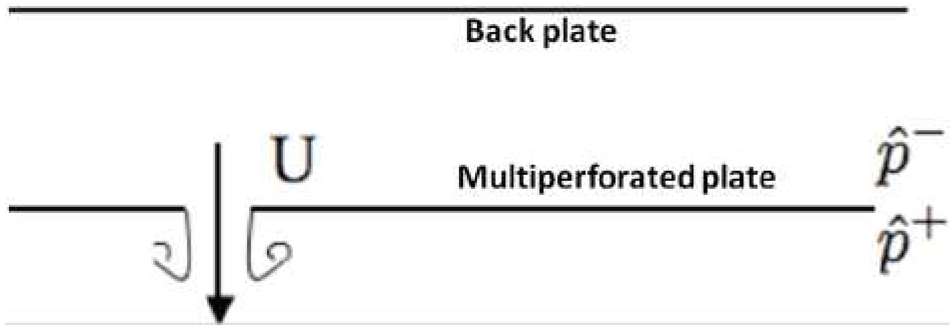
- … but has also an acoustic effect.

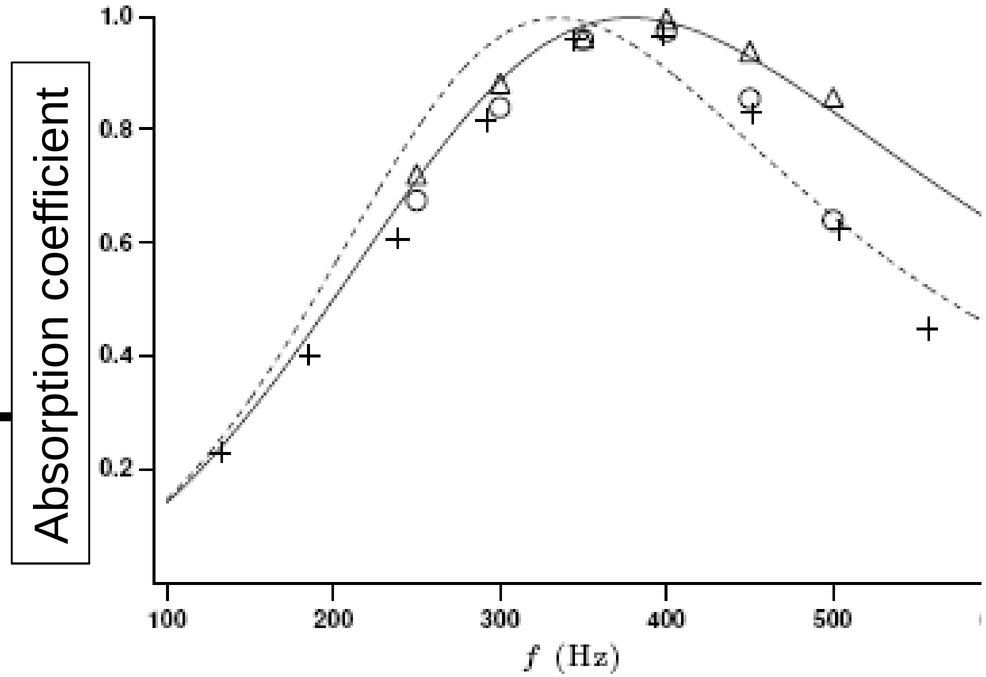
Multiperforated liners

400 Hz

250 Hz





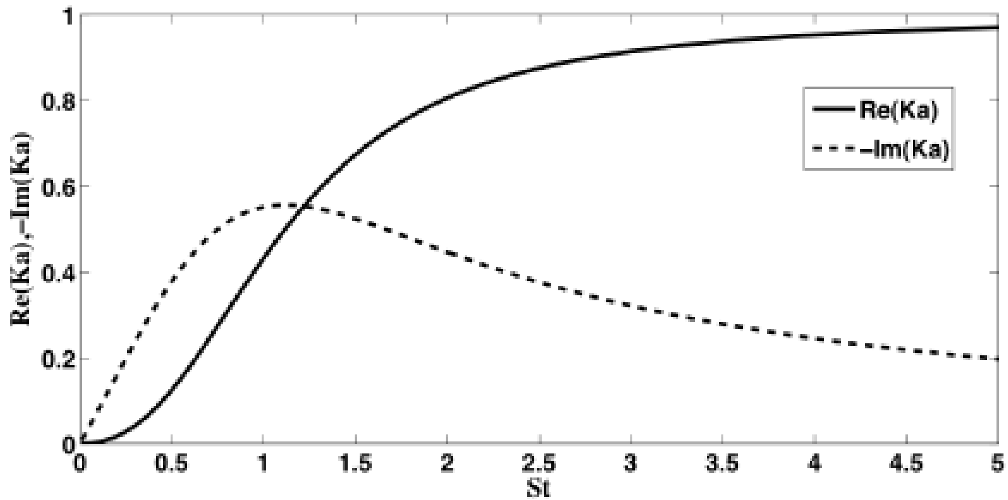


Multiperforated liners

- The Rayleigh conductivity (Howe 1979)

$$K_R = \frac{j\ \omega\ \rho_0 d^2 \widehat{u}^{\pm}}{\widehat{p}^+ - \widehat{p}^-}.$$

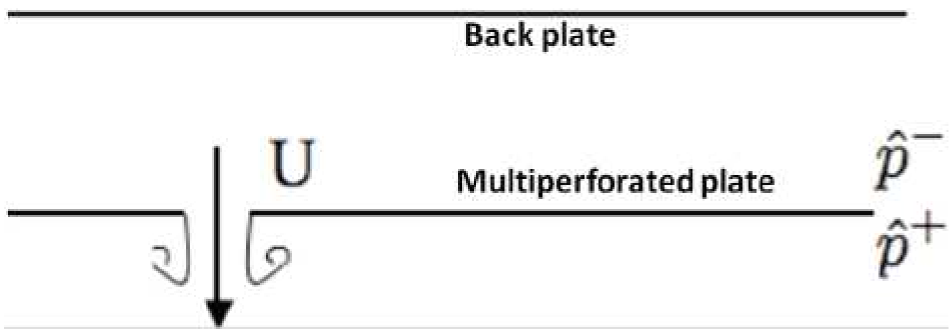
$$St = \frac{\omega a}{U}$$



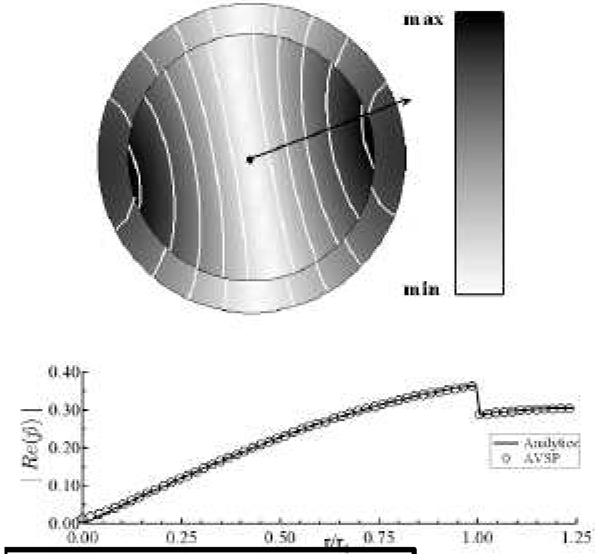
- Under the M=0 assumption:

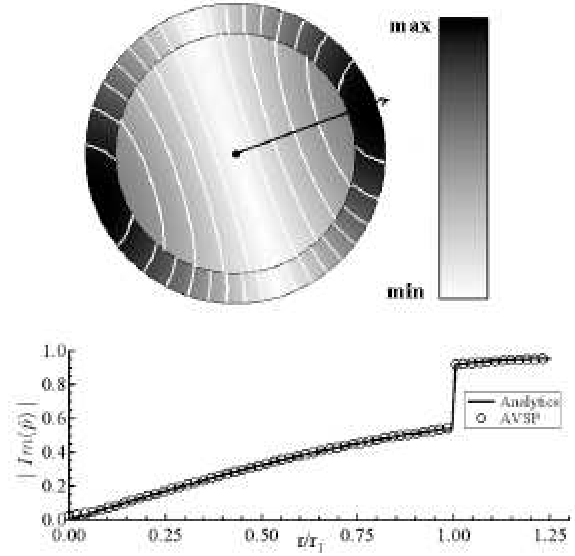
$$\nabla \widehat{p} \cdot \mathbf{n} = \frac{K_R}{d^2}[\widehat{p}^+ - \widehat{p}^-]$$

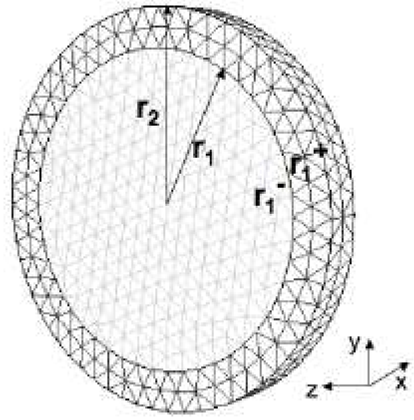
(impedance-like condition)



Academic validation

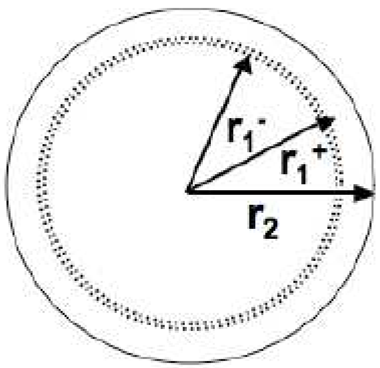


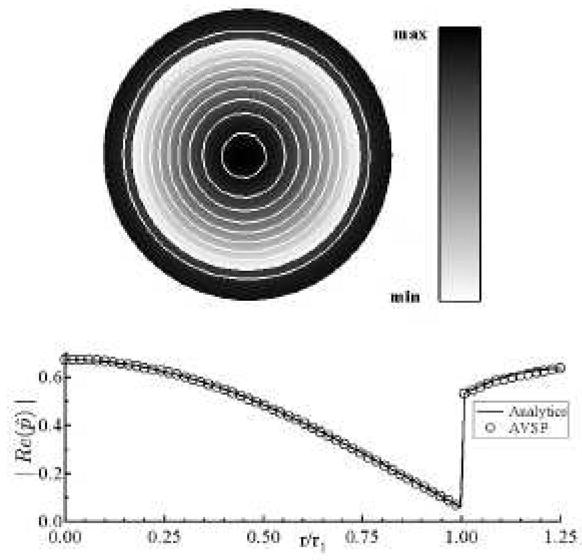


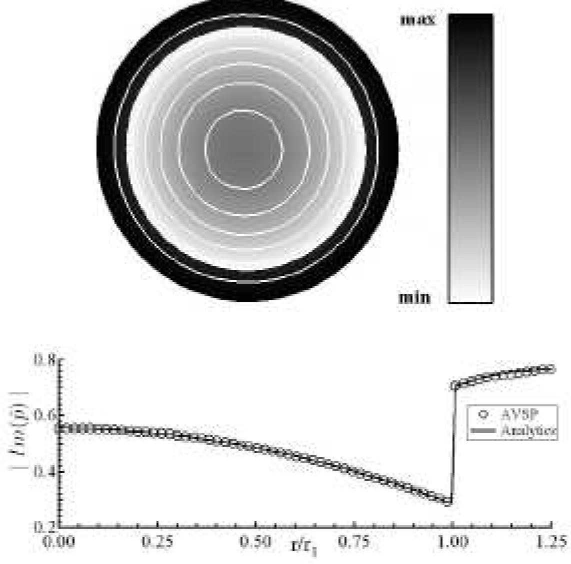


$$f = 382 - 18i \text{ Hz}$$

$$f = 534 - 97i \text{ Hz}$$







Question

- There are many modes in the low-frequency regime
- They can be predicted in complex geometries
- Boundary conditions and multiperforated liners have first order effect and they can be accounted for properly
- All these modes are potentially dangerous

Which of these modes are made unstable by the flame ?

Accounting for the unsteady flame

- Need to solve the thermo-acoustic problem

$$\boxed{\gamma p_0 \nabla \cdot \left(\frac{1}{\rho_0} \vec{\nabla} \hat{p} \right) + \omega^2 \hat{p} = j\omega(\gamma - 1)\hat{q}}$$

$$\text{e.g.}: \; j\omega(\gamma-1)\hat{q} = \frac{(\gamma\text{-}1)\, q_{\text{mean}}}{\rho_{0,x_{ref}} U_{\text{bulk}}} n_l(\mathbf{x},\omega) e^{j\Re(\omega)\tau_l(\mathbf{x},\omega)} \nabla \hat{p}_{\mathbf{x}_{\text{ref}}} \cdot \mathbf{n}_{\text{ref}} \qquad \text{" local } FTF \text{ model"}$$

- In discrete form

$$\mathbf{AP} + \omega \mathbf{BP} + \omega^2 \mathbf{CP} = \mathbf{D}(\omega)\mathbf{P}$$

$$\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D} : \text{square matrices}$$

Non-Linear Eigenvalue Problem of size N

Iterative method

1. Solve the Quadratic EVP

$$\mathbf{AP} + \omega_0 \mathbf{BP} + \omega_0^2 \mathbf{CP} = 0$$

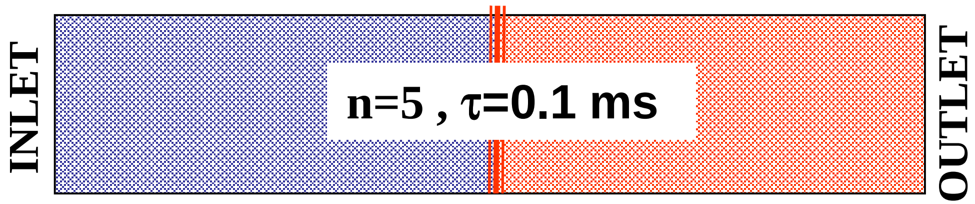
2. At iteration *k*, solve the Quadratic EVP

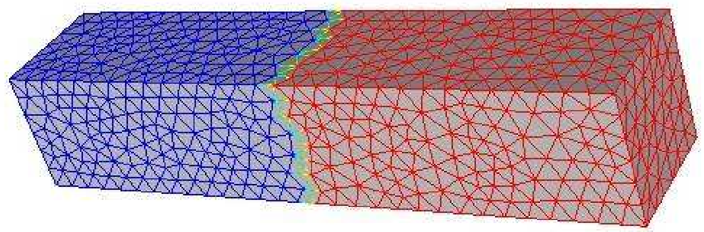
$$\left[\mathbf{A} - \mathbf{D}(\omega_{k-1})\right]\mathbf{P} + \omega_k \mathbf{BP} + \omega_k^2 \mathbf{CP} = 0$$

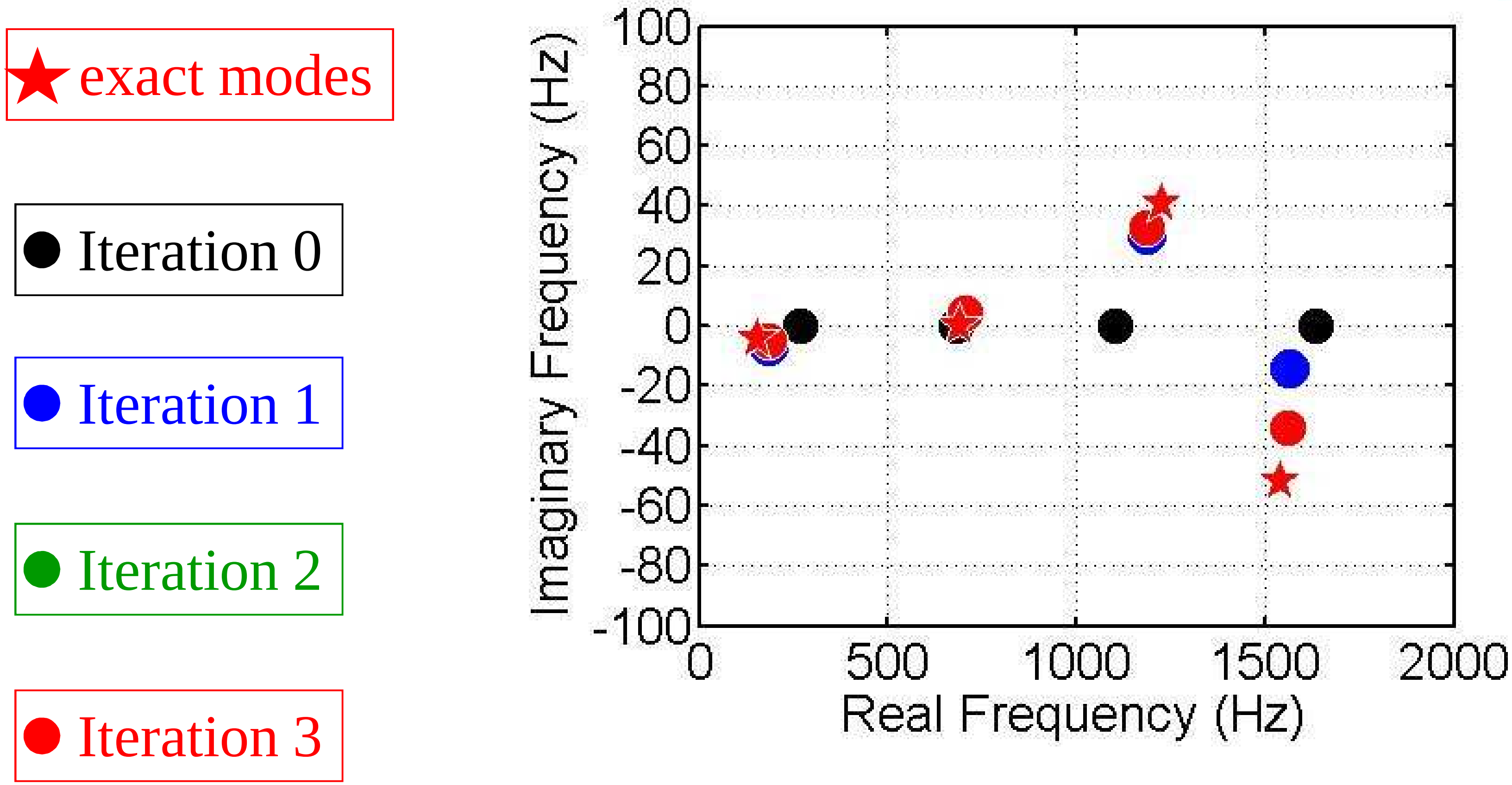
3. Iterate until convergence $\left|\omega_k - \omega_{k-1}\right| < \mathrm{tol}$

$\omega_k, \mathbf{P}$ is solution of the thermo-acoustic problem

Comparison with analytic results







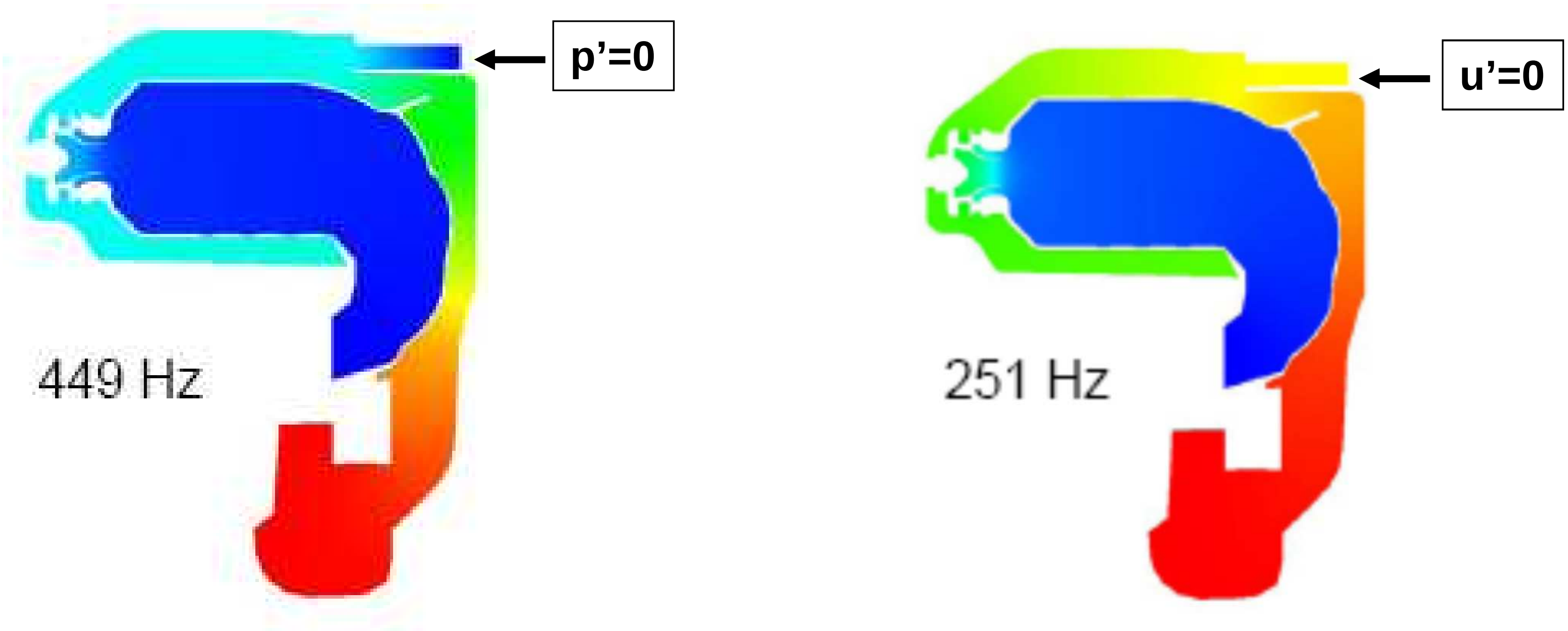
About the iterative method

- No general prove of convergence can be given except for academic cases
- If it converges, the procedure gives the exact solution of the discrete thermo-acoustic problem
- The number of iterations must be kept small for efficiency (one Quadratic EVP at each step and for each mode)
- Following our experience, the method does converge in a few iterations … in most cases !!

OUTLINE

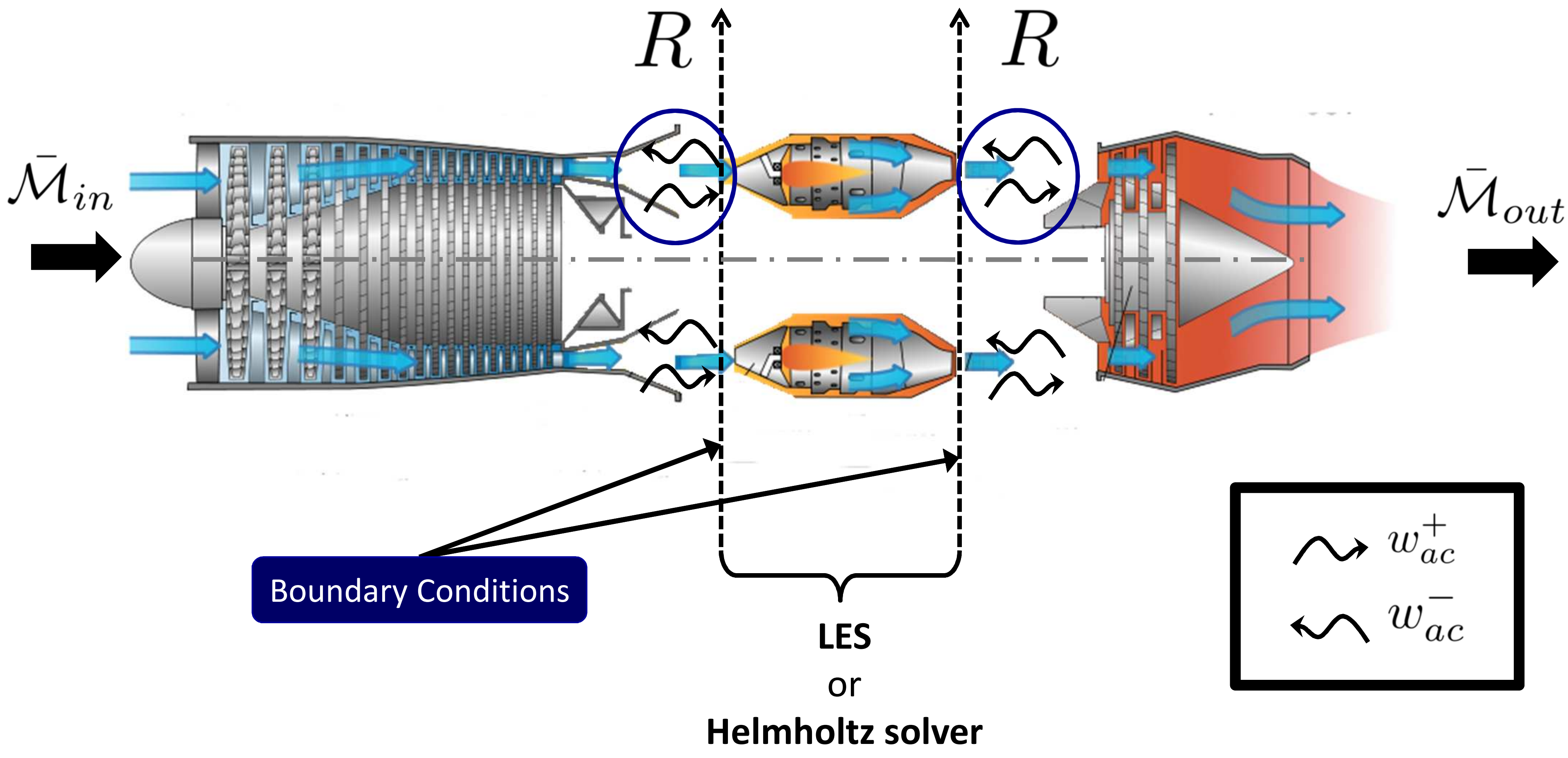
1. Computing the whole flow
2. Computing the fluctuations
3. Boundary conditions
4. Analysis of an annular combustor

BC essential for thermo-acoustics



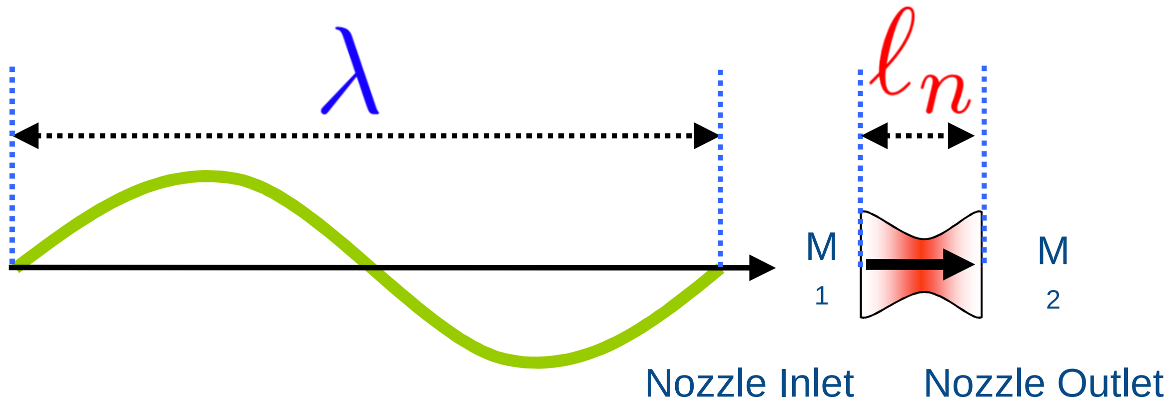
Acoustic analysis of a Turbomeca combustor including the swirler, the casing and the combustion chamber
C. Sensiau (CERFACS/UM2) – AVSP code

Acoustic boundary conditions



Analytical approach

• Assumption of quasi-steady flow (low frequency)

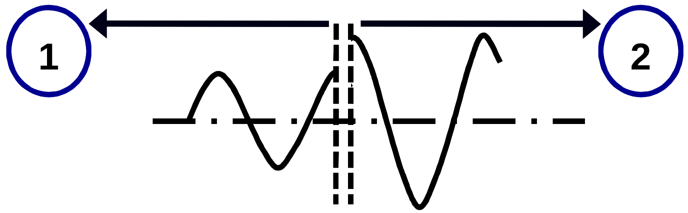


$$\lambda \gg \ell_n$$

• Theories (e.g.: Marble & Candle, 1977; Cumpsty & Marble, 1977; Stow et al., 2002; …)

• Conservation of Total temperature, Mass flow, Entropy fluctuations

Analytical approach



Mass

$$\bar{\rho}_1 \hat{u}_1 + \hat{\rho}_1 \bar{u}_1 = \bar{\rho}_2 \hat{u}_2 + \hat{\rho}_2 \bar{u}_2$$

Momentum

$$\hat{p}_1 + \hat{\rho}_1 \bar{u}_1^2 + 2\bar{\rho}_1 \bar{u}_1 \hat{u}_1 + \hat{\mathcal{F}} = \hat{p}_2 + \hat{\rho}_2 \bar{u}_2^2 + 2\bar{\rho}_2 \bar{u}_2 \hat{u}_2$$

Energy

$$c_p \bar{T}_{t1}(\bar{\rho}_1 \hat{u}_1 + \hat{\rho}_1 \bar{u}_1) + \bar{\rho}_1 \bar{u}_1 (c_p \hat{T}_1 + \bar{u}_1 \hat{u}_1) + \hat{\mathcal{W}} = c_p \bar{T}_{t2}(\bar{\rho}_2 \hat{u}_2 + \hat{\rho}_2 \bar{u}_2) + \bar{\rho}_2 \bar{u}_2 (c_p \hat{T}_2 + \bar{u}_2 \hat{u}_2)$$

Analytical approach

- w^+ Acoustic wave
- w^- Acoustic wave
- w^s Entropy wave

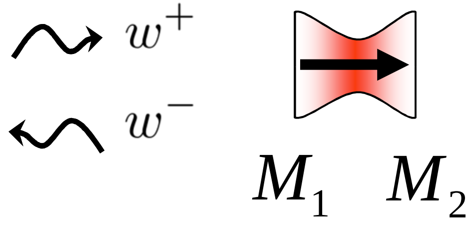
Compact systems

$$\frac{\hat{p}}{\gamma \bar{p}} = \frac{1}{2}\hat{w}^+ + \frac{1}{2}\hat{w}^- \approx \frac{1}{2}|\hat{w}^+| + \frac{1}{2}|\hat{w}^-|$$

$$\frac{\hat{u}}{\bar{c}} = \frac{1}{2}\hat{w}^+ - \frac{1}{2}\hat{w}^- \approx \frac{1}{2}|\hat{w}^+| - \frac{1}{2}|\hat{w}^-|$$

$$\frac{\hat{s}}{c_p} = \hat{w}^S \approx |\hat{w}^S|$$

Example: Compact nozzle



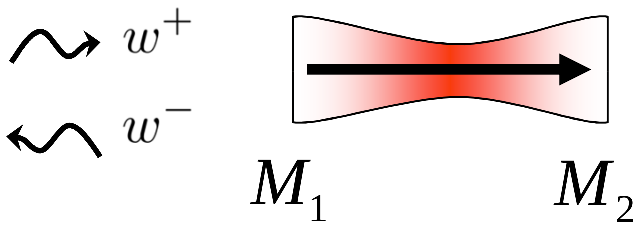
1. Compact choked nozzle:

$$\frac{\omega^-}{\omega^+} = \frac{1-(\gamma-1)M_1/2}{1+(\gamma-1)M_1/2}$$

2. Compact unchoked nozzle:

$$\frac{\omega^-}{\omega^+} = \frac{(1+M_1)\left[M_2\left(1+(\gamma-1)M_1^2/2\right)-M_1\left(1+(\gamma-1)M_2^2/2\right)\right]}{(1-M_1)\left[M_2\left(1+(\gamma-1)M_1^2/2\right)+M_1\left(1+(\gamma-1)M_2^2/2\right)\right]}$$

Non compact elements



The proper equations in the acoustic element are the quasi 1D LEE:

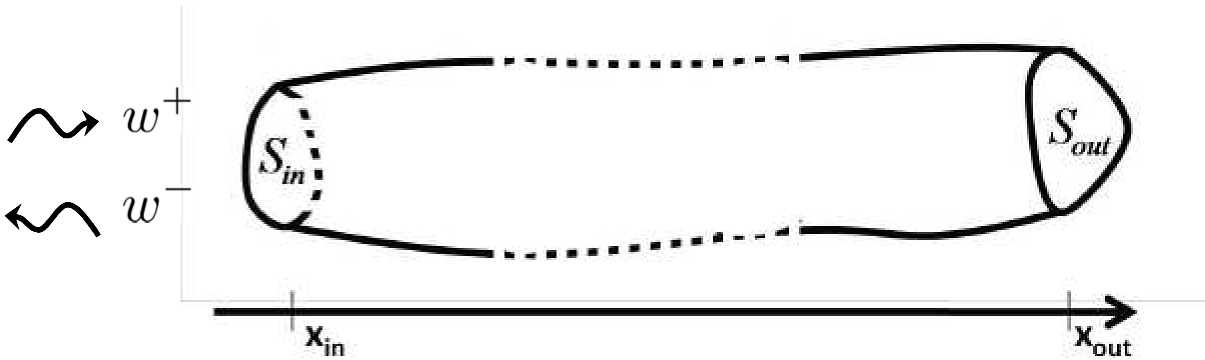
$$\left(\frac{\partial \bar{u}}{\partial x} + \bar{u}\frac{\partial}{\partial x} + \frac{\bar{u}}{\mathcal{S}}\frac{\partial \mathcal{S}}{\partial x}\right)\hat{\rho} + \left(\frac{\partial \bar{\rho}}{\partial x} + \bar{\rho}\frac{\partial}{\partial x} + \frac{\bar{\rho}}{\mathcal{S}}\frac{\partial \mathcal{S}}{\partial x}\right)\hat{u} - j\omega\hat{\rho} = 0$$

$$\left(\frac{1}{\bar{\rho}}\frac{\partial \bar{c}^2}{\partial x} + \frac{\bar{u}}{\bar{\rho}}\frac{\partial \bar{u}}{\partial x} + \frac{\bar{c}^2}{\bar{\rho}}\frac{\partial}{\partial x}\right)\hat{\rho} + \left(\frac{\partial \bar{u}}{\partial x} + \bar{u}\frac{\partial}{\partial x}\right)\hat{u} + (\gamma - 1)\bar{T}\left(\frac{1}{\bar{p}}\frac{\partial \bar{p}}{\partial x} + \frac{\partial}{\partial x}\right)\hat{s} + \hat{F} - j\omega\hat{u} = 0$$

$$\frac{\partial \bar{s}}{\partial x}\hat{u} + \bar{u}\frac{\partial \hat{s}}{\partial x} - \hat{\mathcal{W}}_k - j\omega\hat{s} = 0$$

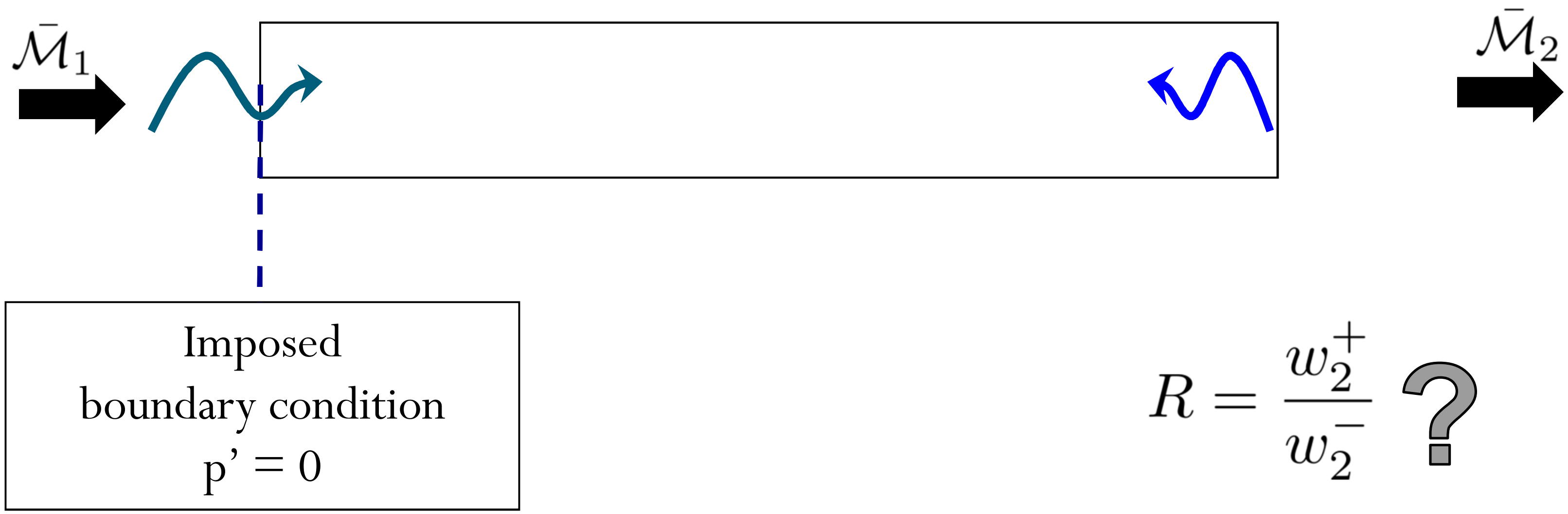
Due to compressor or turbine

Principle of the method

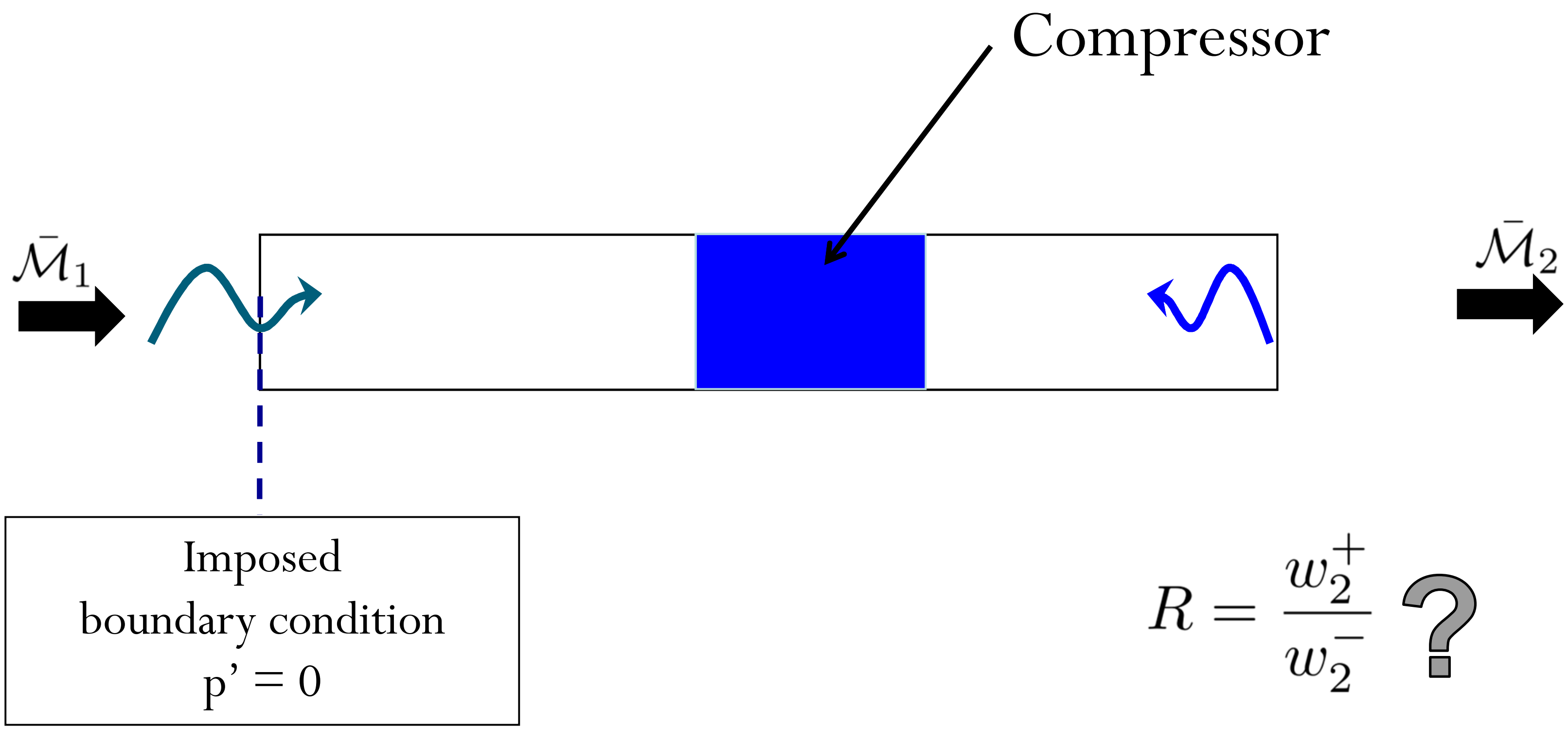


1. The boundary condition is well known at x_{out} (e.g.: p'=0)
2. Impose a non zero incoming wave at x_{in}
3. Solve the LEE in the frequency space
4. Compute the outgoing wave at x_{in}
5. Deduce the effective reflection coefficient at x_{in}

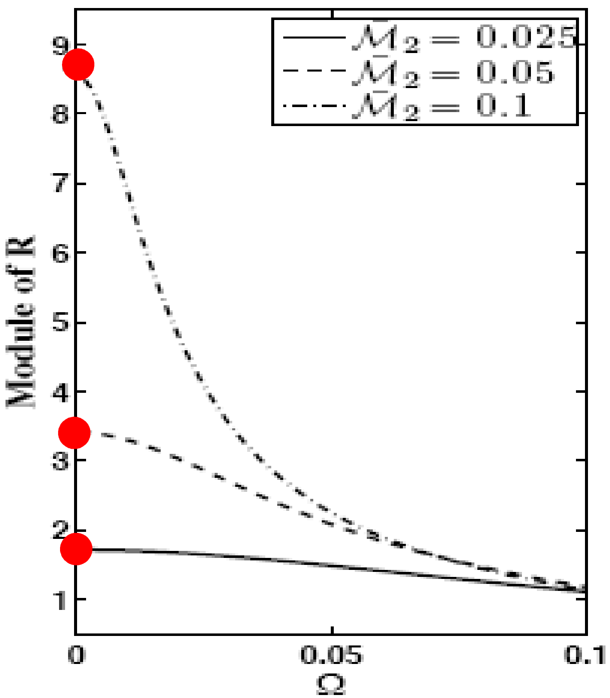
Example of a ideal compressor



Example of a ideal compressor

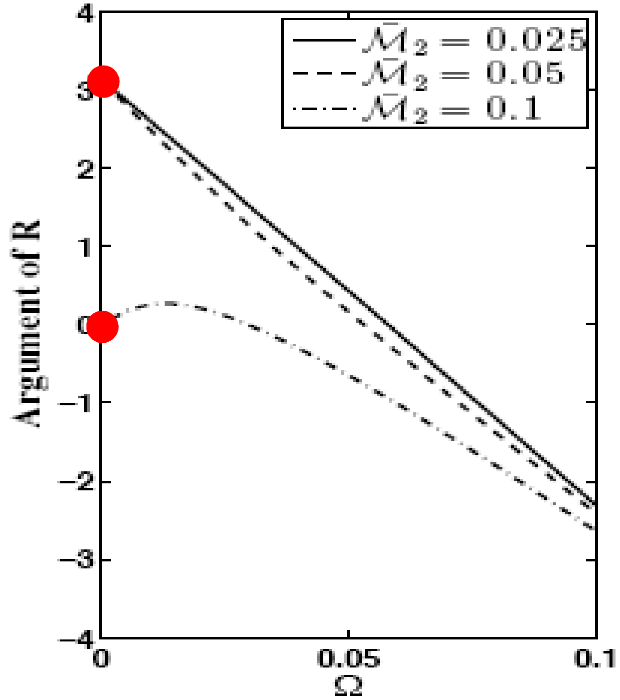


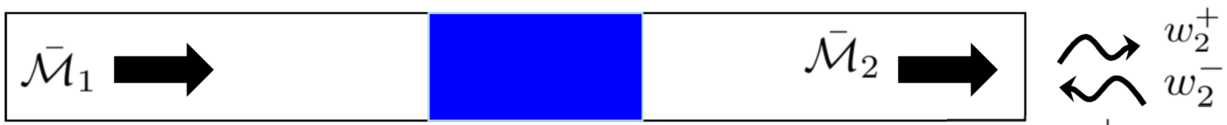
Example of a ideal compressor



$$\bar{\pi}_c = \frac{p_{t2}}{p_{t1}} = 4$$

● **Compact theory**

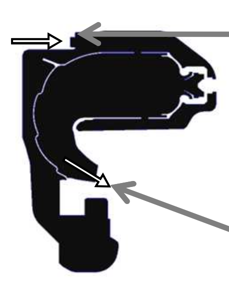




$p' = 0$

$$R = \frac{w_2^+}{w_2^-}$$

Realistic air intake



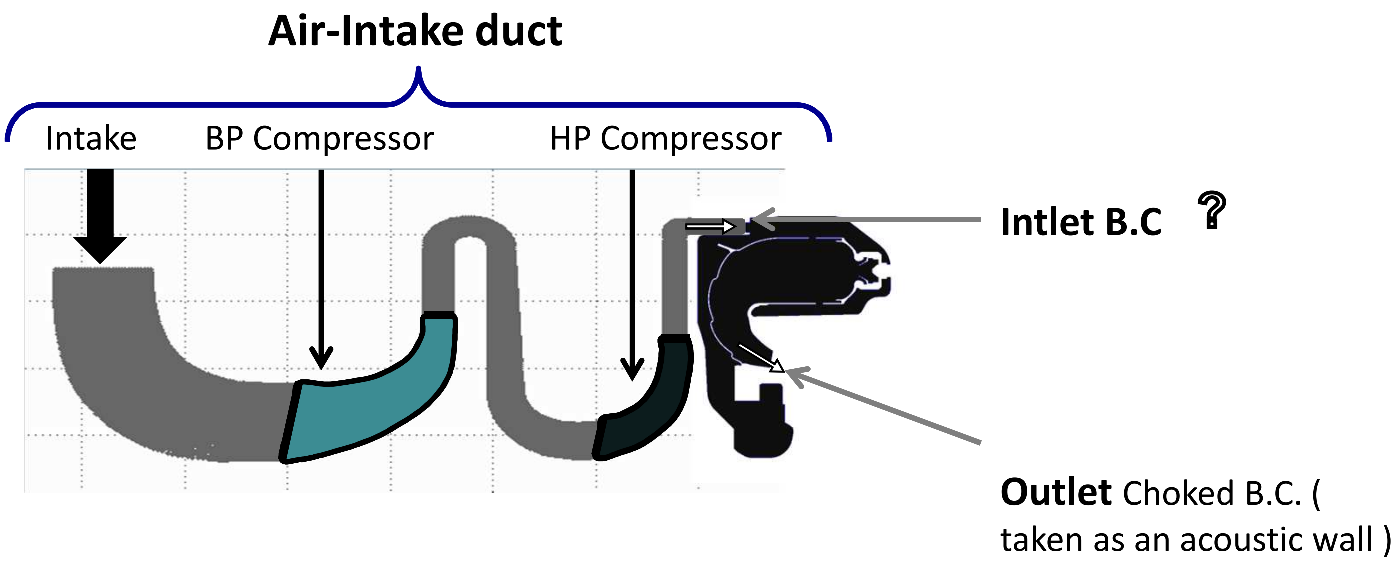
Intlet B.C ?

Outlet Choked B.C. (
taken as an acoustic wall)

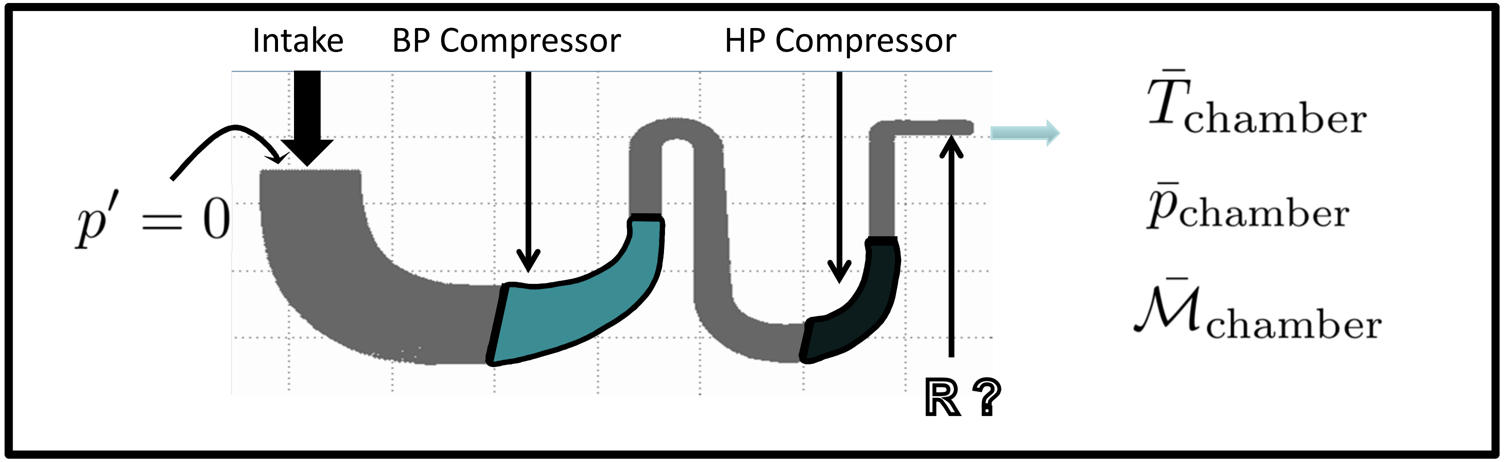
Acoustic modes of the
combustion chamber

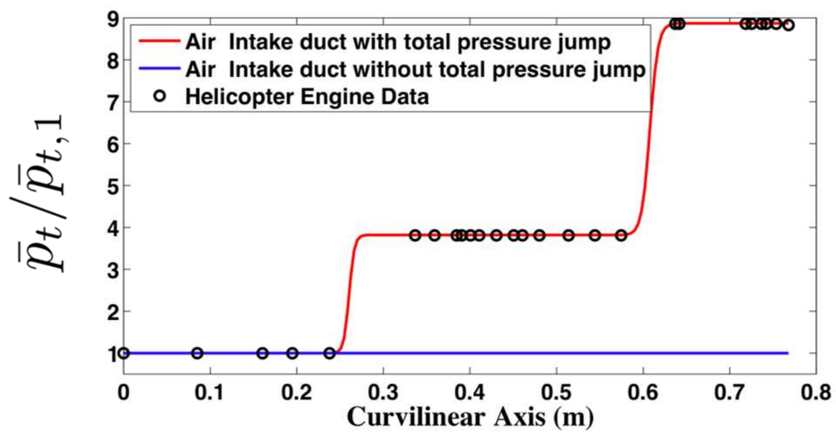
Need to be computed

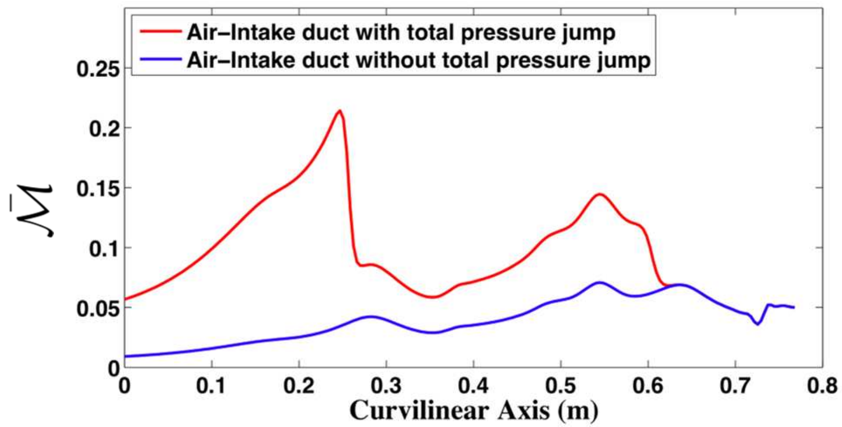
Realistic air intake



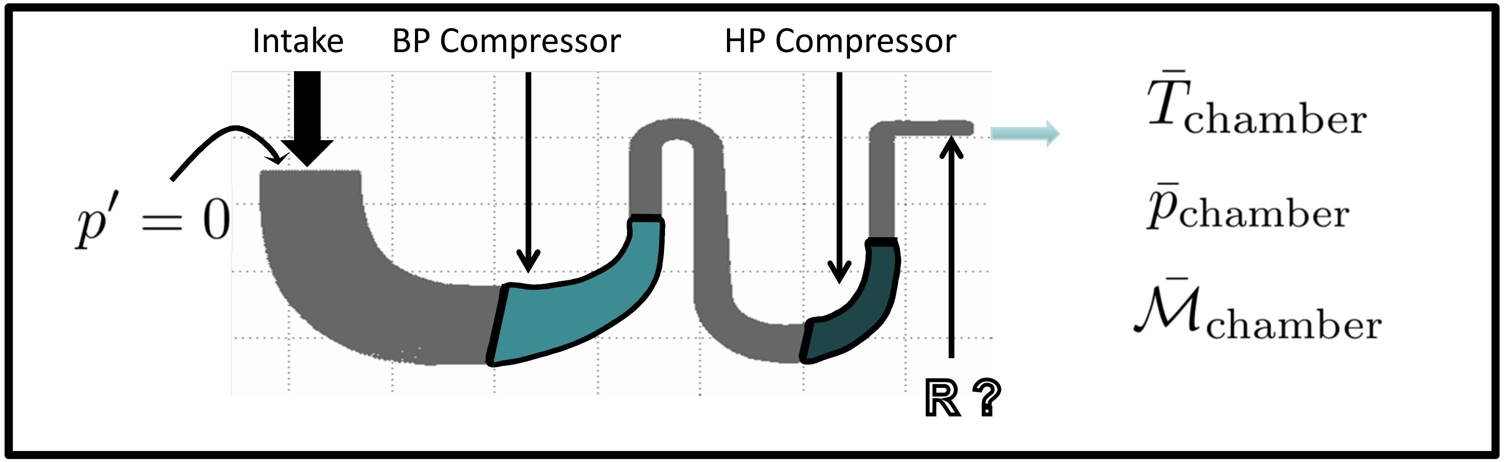
Realistic air intake

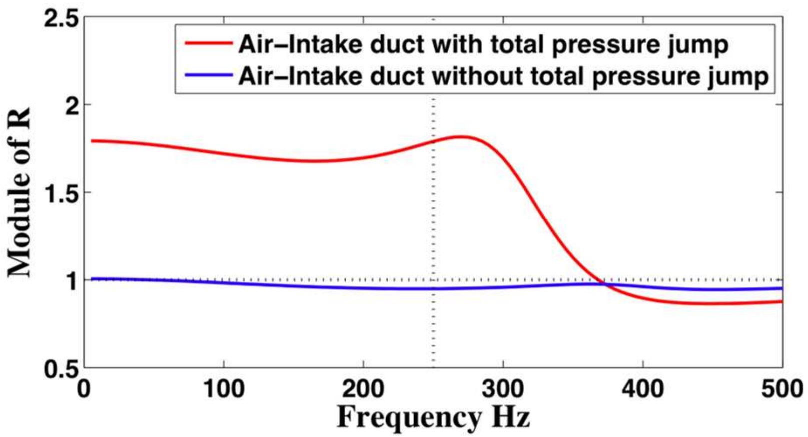


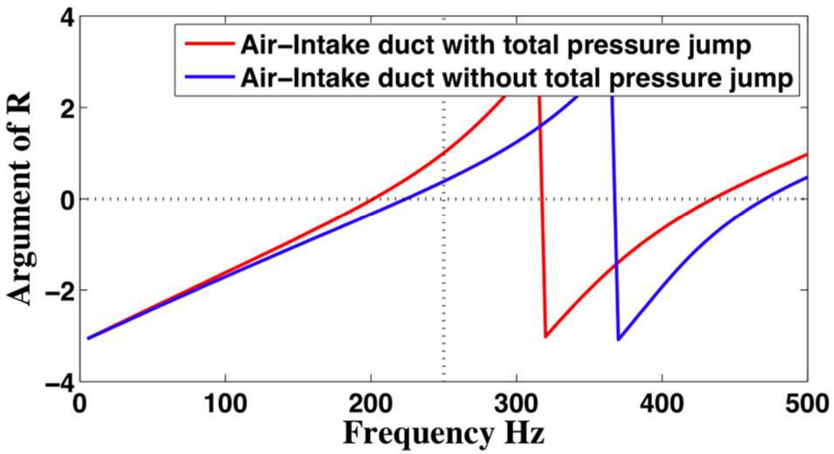




Realistic air intake





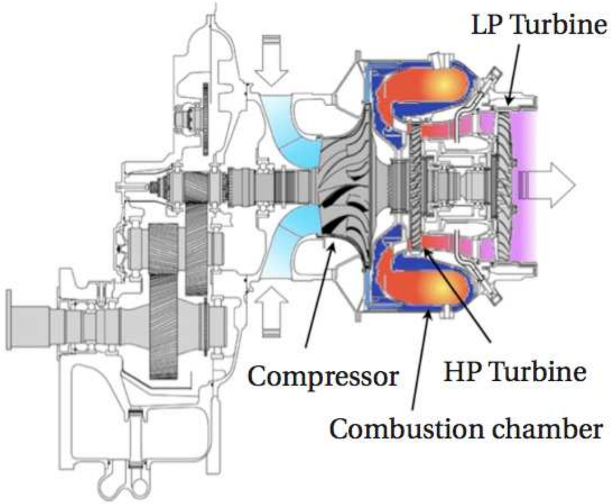


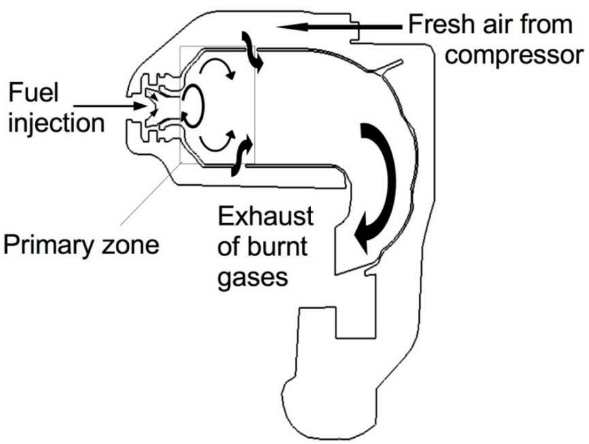
OUTLINE

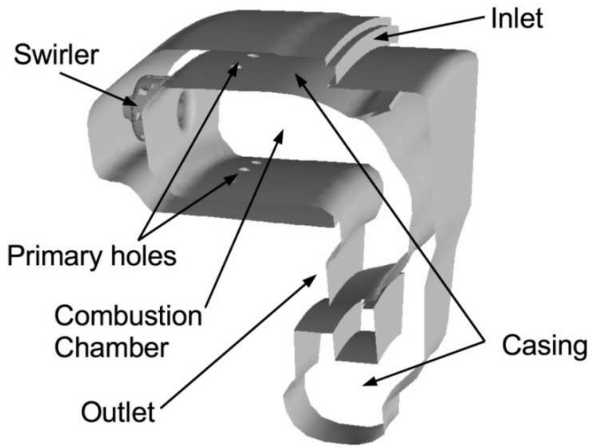
1. Computing the whole flow
2. Computing the fluctuations
3. Boundary conditions
4. Analysis of an annular combustor

Overview of the configuration

- Helicopter engine
- 15 burners
- From experiment: 1A mode may run unstable



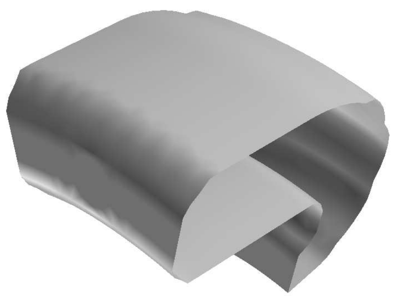




About the computational domain

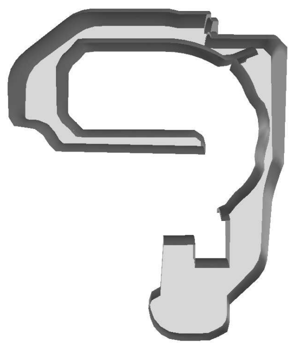
- When dealing with actual geometries, defining the computational domain may be an issue
- Turbomachinery are present upstream/downstream
- The combustor involves many "details": combustion chamber, swirler, casing, primary holes, multi-perforated liner – Dassé et al., AIAA 2008-3007, …

Combustion Chamber



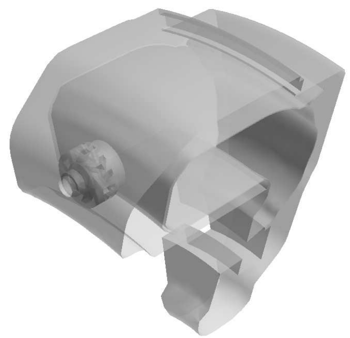
700 Hz

Casing



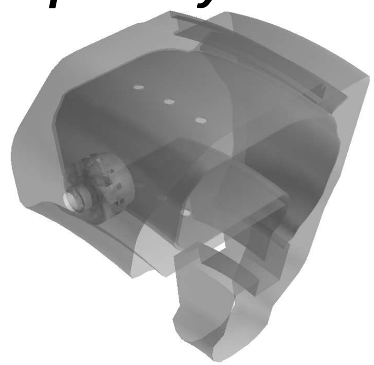
500 Hz

CC + casing + swirler



575 Hz

CC + casing + swirler + primary holes



609 Hz

Frequency of the first azimuthal mode – 1A

Is this mode stable ?

- The acoustic mode found at 609 Hz has a strong azimuthal component, like the experimentally observed instability

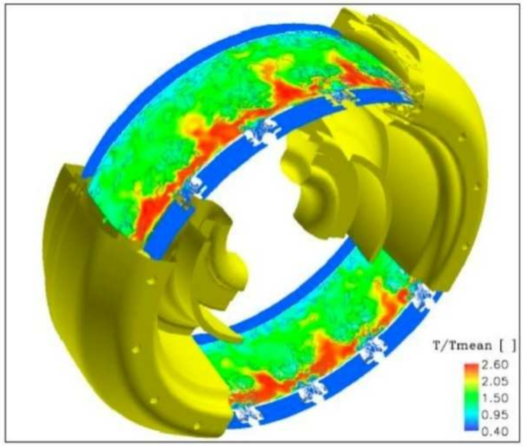
- Its stability can be assessed by solving the thermo-acoustic problem which includes the flame response

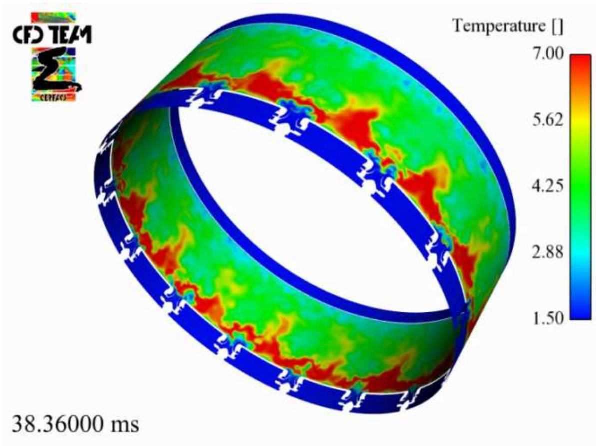
- In this annular combustor, there are 15 turbulent flames …

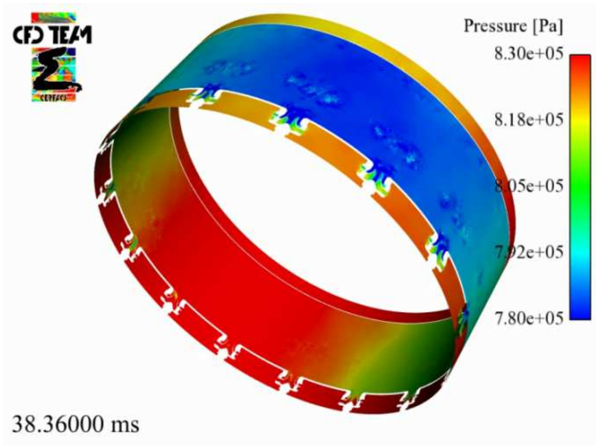
- Do they share the same response ?

Using the brute force …

Large Eddy Simulation of the full annular combustion chamber Staffelbach et al., *2008*



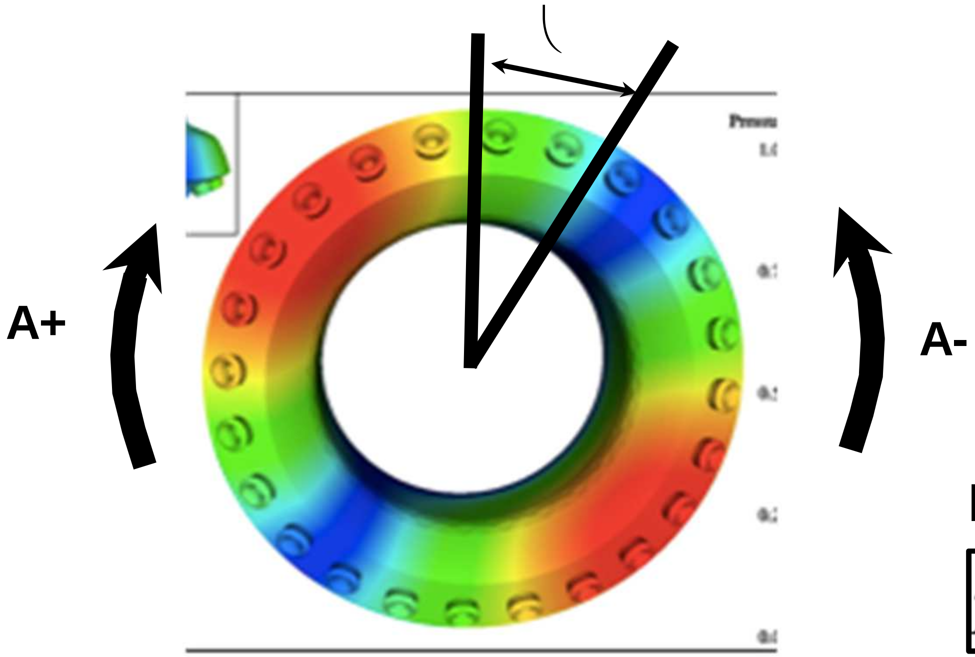




- The first azimuthal mode is found unstable from LES, at 740 Hz
- Same mode found unstable experimentally

Mode structure

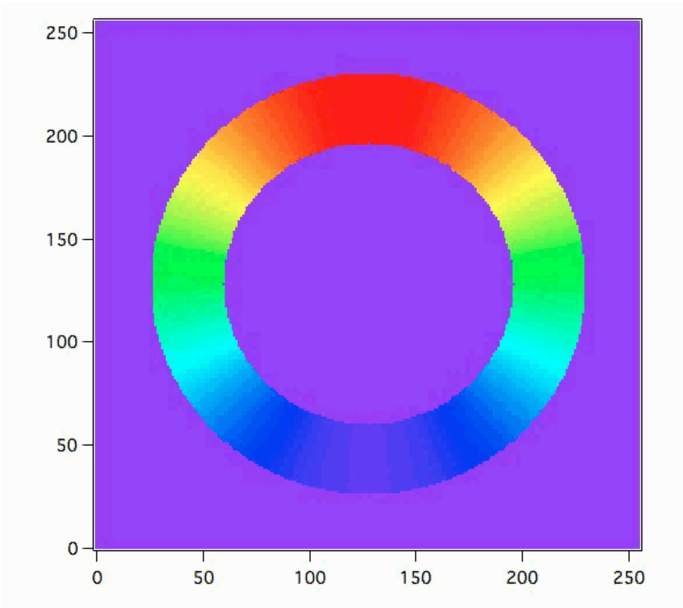
- In annular geometries, two modes may share the same frequency
- These two modes may be of two types
- Consider the pressure as a function of the angular position

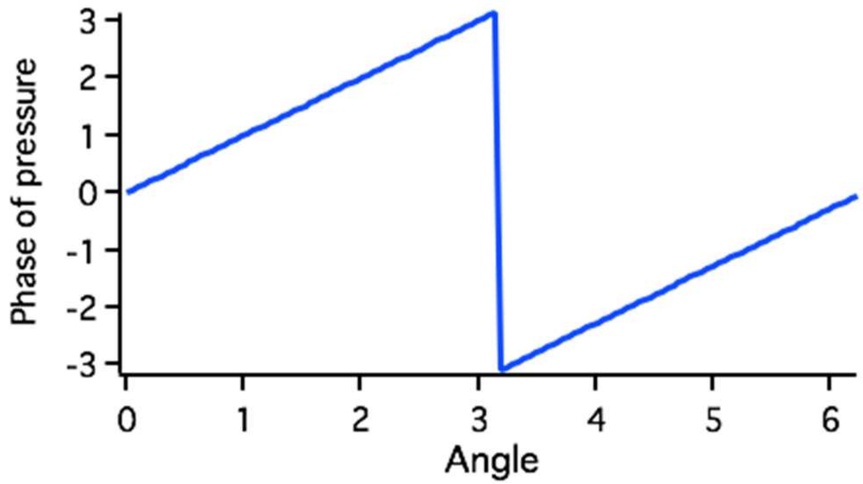


From simple acoustics:

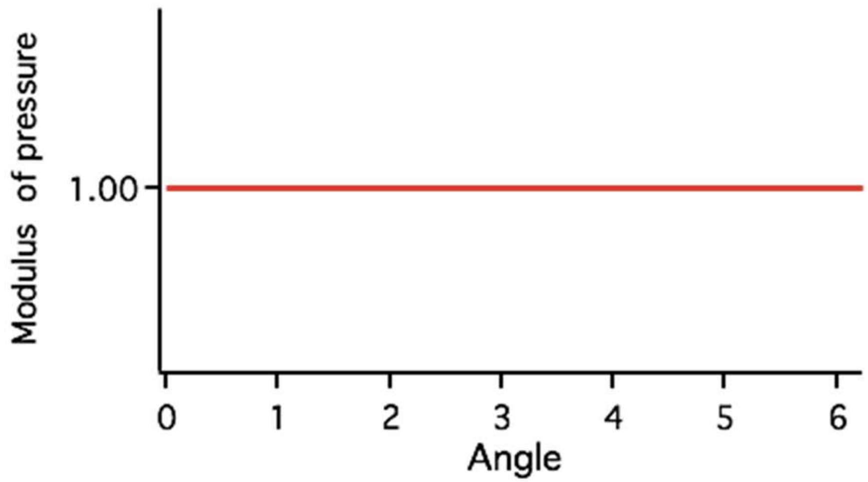
$$p' = \left[A_+ e^{i\theta} + A_- e^{-i\theta}\right] e^{-i\omega t}$$

Turning mode. A+ = 1 & A- = 0

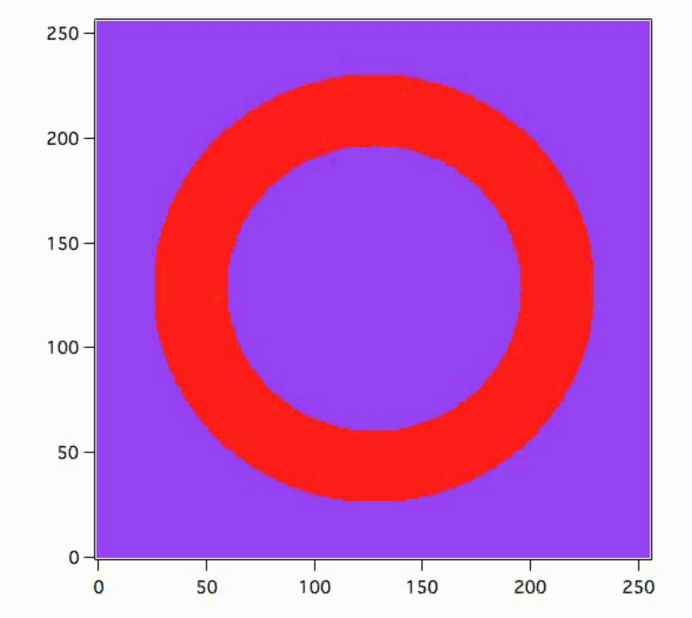


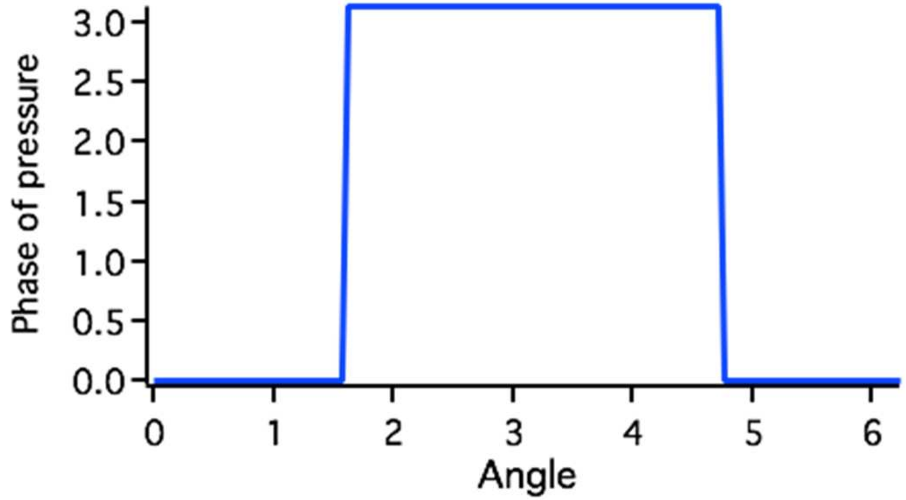


Companion mode turns counter clockwise

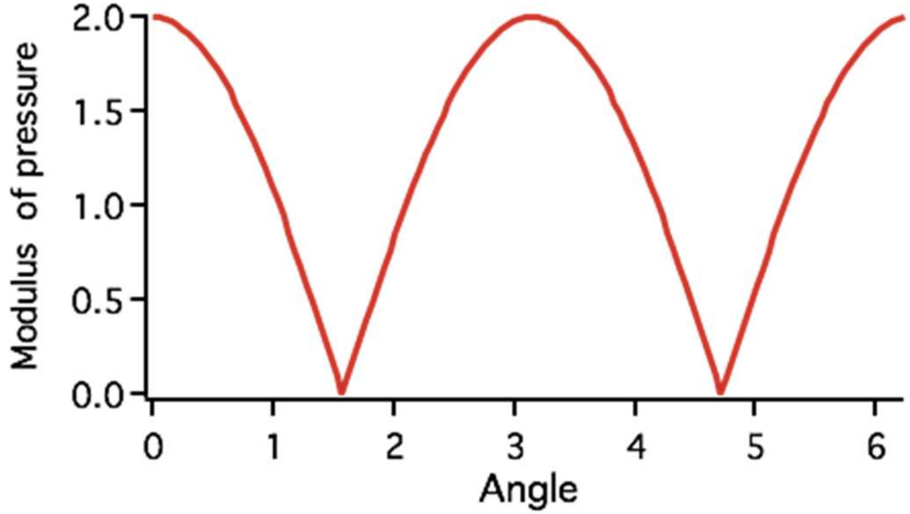


Standing mode. A+ = A- = 1

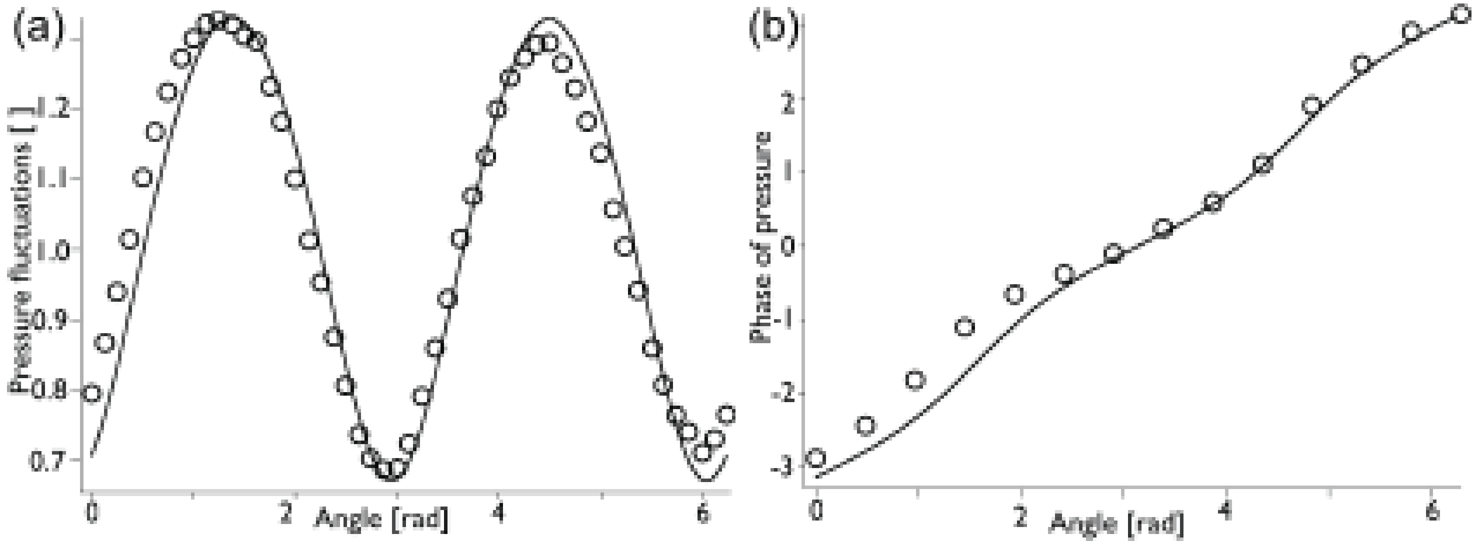




Companion mode has its nodes/antinodes at opposite locations



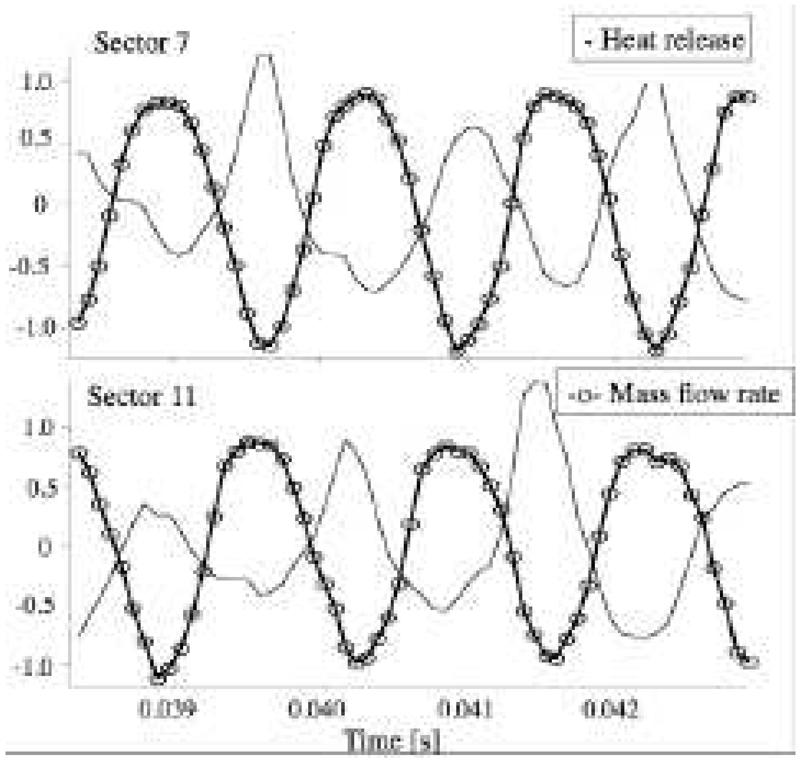
LES mode. A+ = 1   A- = 0.3



- A simple analytical model can explain the global mode shape obtained from LES
- Can we predict the stability by using the Helmholtz solver ?

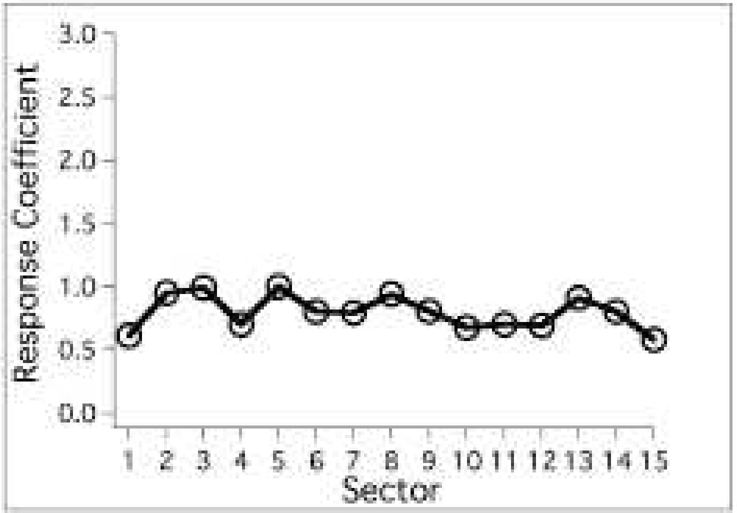
Responses of the burners

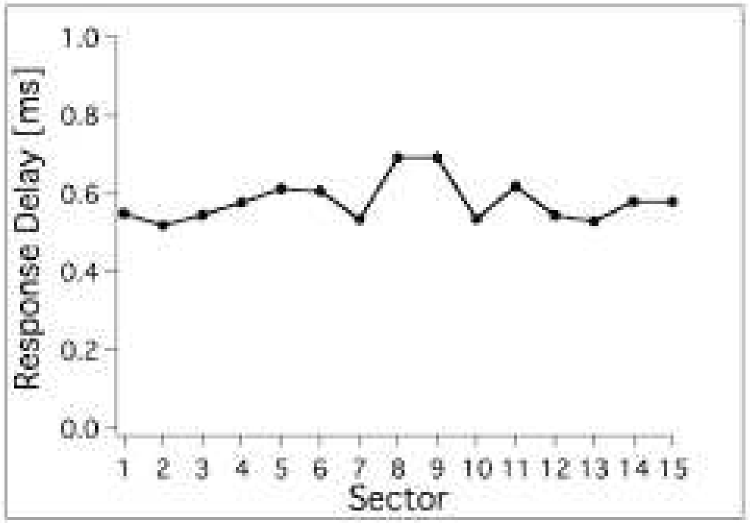
- Because of the self-sustained instability, the pressure in front of each burner oscillates
- This causes flow rate oscillations
- Because the unstable mode is turning, the flow rate through the different burners are not in phase



Global Response of the flames

- From the full LES, the global flame transfer of the 15 burners can be computed from the global HR and the flow rate signals
- For all the 15 sectors, the amplitude of the response is close to 0.8 and the time lag is close to 0.6 ms
- This result support the ISSAC assumption …

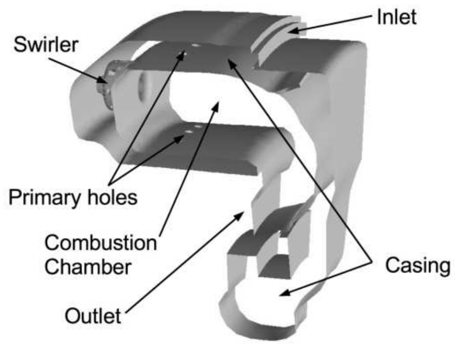




ISAAC Assumption

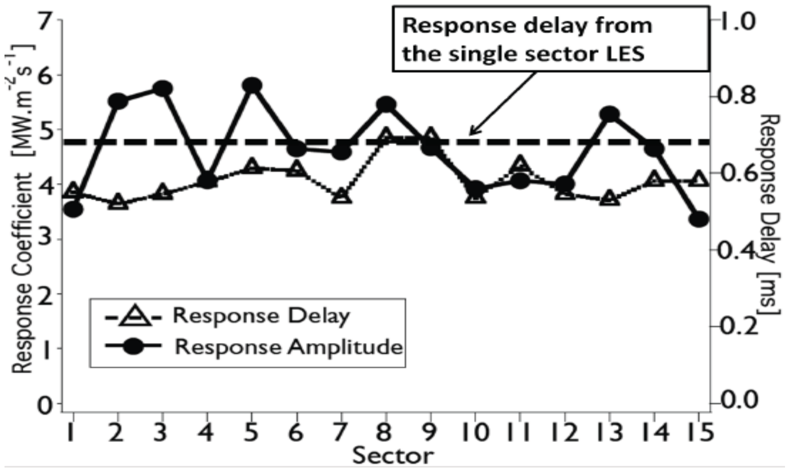
- As far as the flame response is concerned

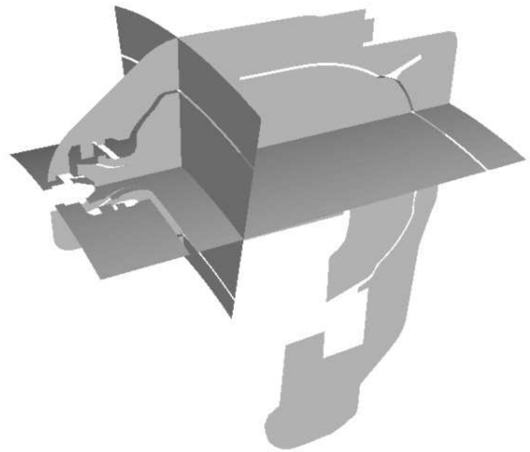
$\Leftrightarrow \quad 15\times$

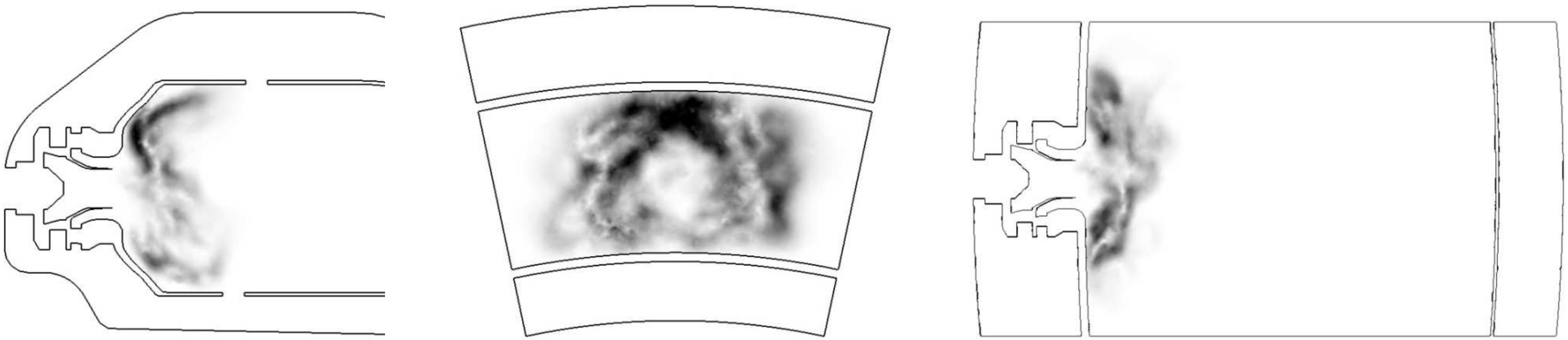


- **I**ndependent **S**ector **A**ssumption for **A**nnular **C**ombustor
- Allows performing a single sector LES with better resolution to obtain more accurate FTF
- Of course the1A mode cannot appear is such computation
- FTF deduced from single sector LES pulsated at 600 Hz

Local flame transfer function







Typical field of interaction index

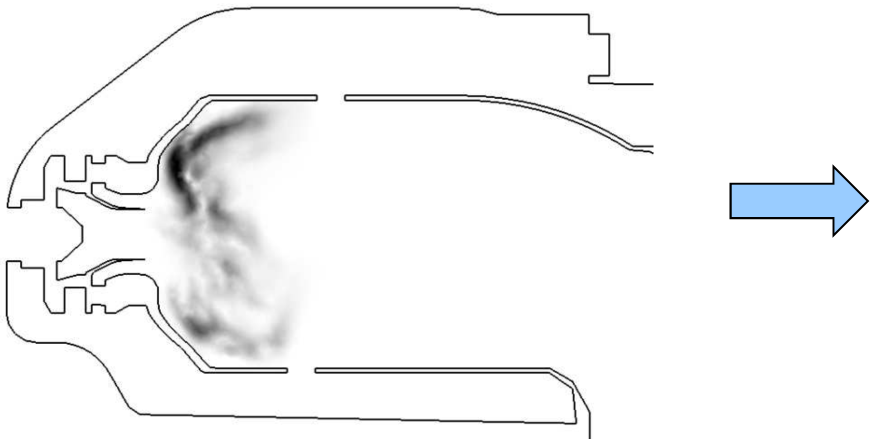
Extended the local n-τ model

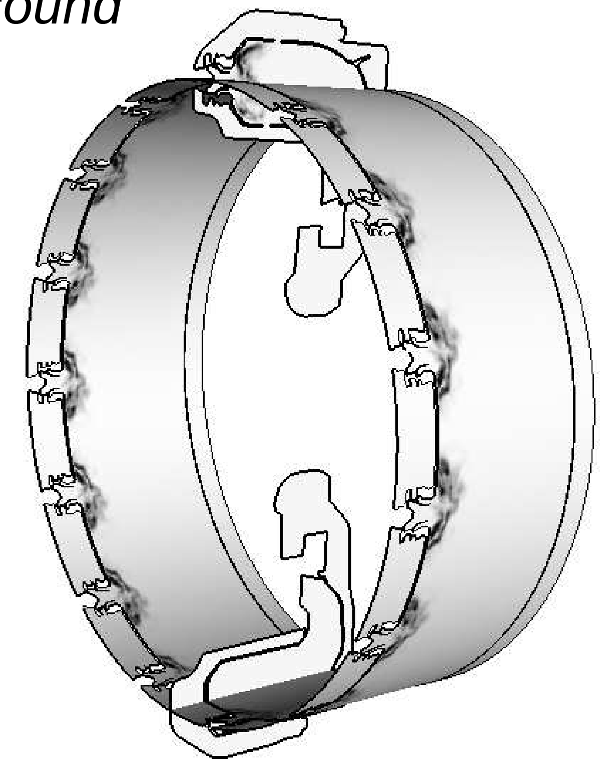
- Define one point of reference upstream of each of the 15 burners
- Use the ISAAC Assumption
- the interaction index $n_l(\mathbf{x},\omega)$, and the time delay $\tau_l(\mathbf{x},\omega)$ are the same in all sectors

$$\hat{\omega}_T(\mathbf{x},\omega) \propto \begin{cases} n_l(\mathbf{x},\omega) \times e^{j\omega\tau_l(\mathbf{x},\omega)} \times \hat{\mathbf{u}}(\mathbf{x}_{\mathrm{ref}}^{1},\omega)\cdot\mathbf{n}_{\mathrm{ref}}^{1} & \text{if } \mathbf{x} \text{ in sector 1} \\ n_l(\mathbf{x},\omega) \times e^{j\omega\tau_l(\mathbf{x},\omega)} \times \hat{\mathbf{u}}(\mathbf{x}_{\mathrm{ref}}^{2},\omega)\cdot\mathbf{n}_{\mathrm{ref}}^{2} & \text{if } \mathbf{x} \text{ in sector 2} \\ \vdots & \\ n_l(\mathbf{x},\omega) \times e^{j\omega\tau_l(\mathbf{x},\omega)} \times \hat{\mathbf{u}}(\mathbf{x}_{\mathrm{ref}}^{14},\omega)\cdot\mathbf{n}_{\mathrm{ref}}^{14} & \text{if } \mathbf{x} \text{ in sector 14} \\ n_l(\mathbf{x},\omega) \times e^{j\omega\tau_l(\mathbf{x},\omega)} \times \hat{\mathbf{u}}(\mathbf{x}_{\mathrm{ref}}^{15},\omega)\cdot\mathbf{n}_{\mathrm{ref}}^{15} & \text{if } \mathbf{x} \text{ in sector 15} \end{cases}$$

Extended the local n-τ model

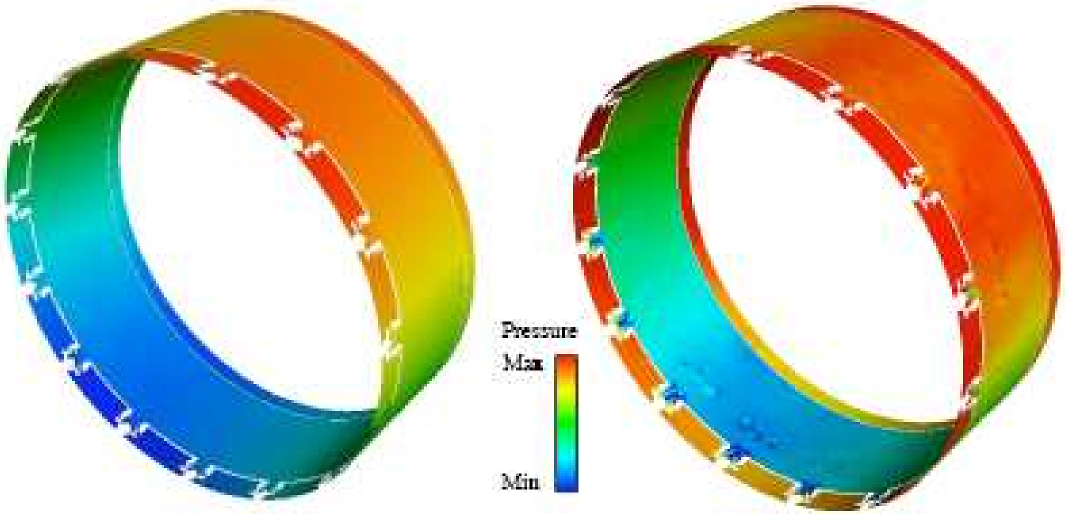
- *Assume* *ISAAC* *and* *duplicate* *the flame transfer function over the whole domain*
- *In the same way, duplicate the field of speed of sound*





Stability of the 1A mode

- The 1A mode from the Helmholtz solver looks like the 1A from the full LES

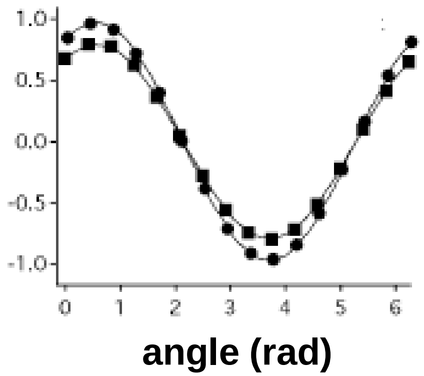


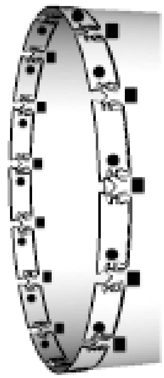
Helmholtz **LES**

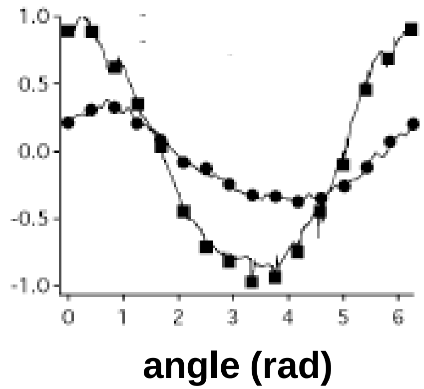
- BUT …
  1. Its frequency remains close to 600 Hz instead of 740 Hz
  2. It is found stable !

Shape of the 1A mode

- The 1A modes from the Helmholtz and LES solvers resemble …
- But a closer look reveals important differences





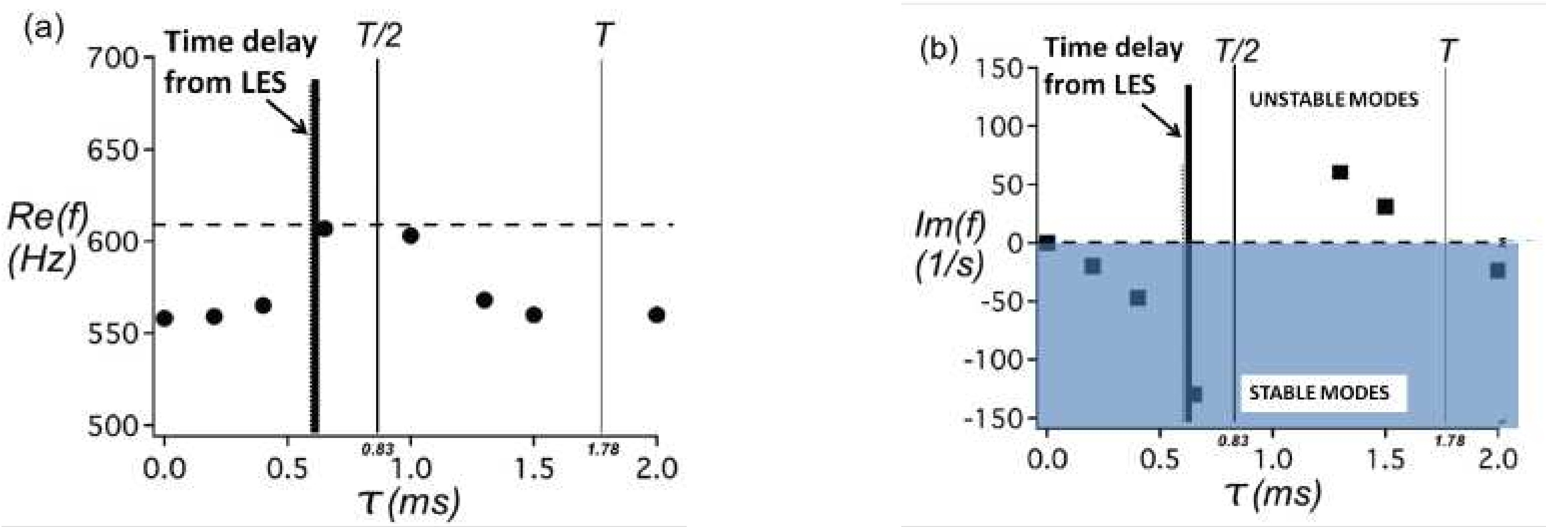


Helmholtz solver **LES solver**

- What is observed in the Helmholtz solver
  1. Pressure amplitude (slightly) larger upstream of the burners
  2. No phase shift between upstream and downstream

What's wrong ?

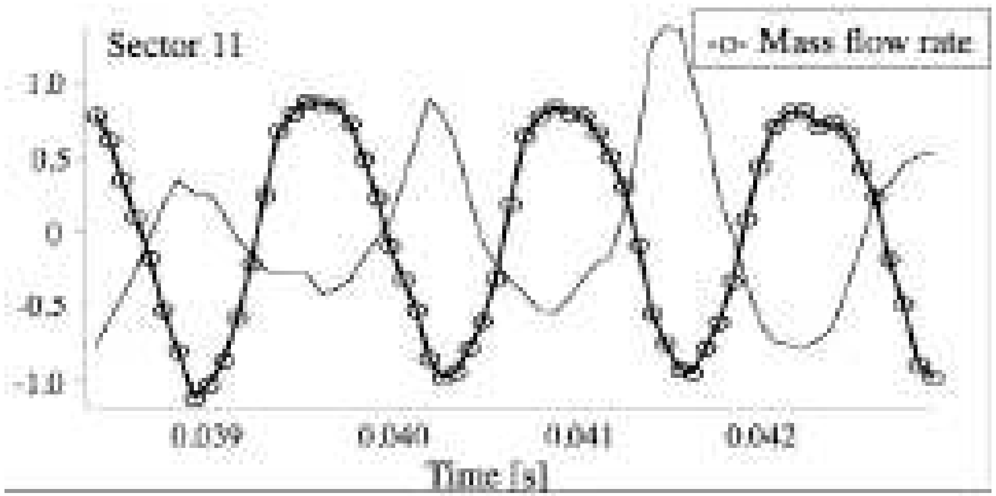
- The Helmholtz solver is not as CPU demanding and allows parametric studies



- The 1A mode is found stable for time delays smaller than T/2
- At 600 Hz, this corresponds to $\tau < 0.83$ ms
- The time delay of the FTF (0.65ms) is below this critical value

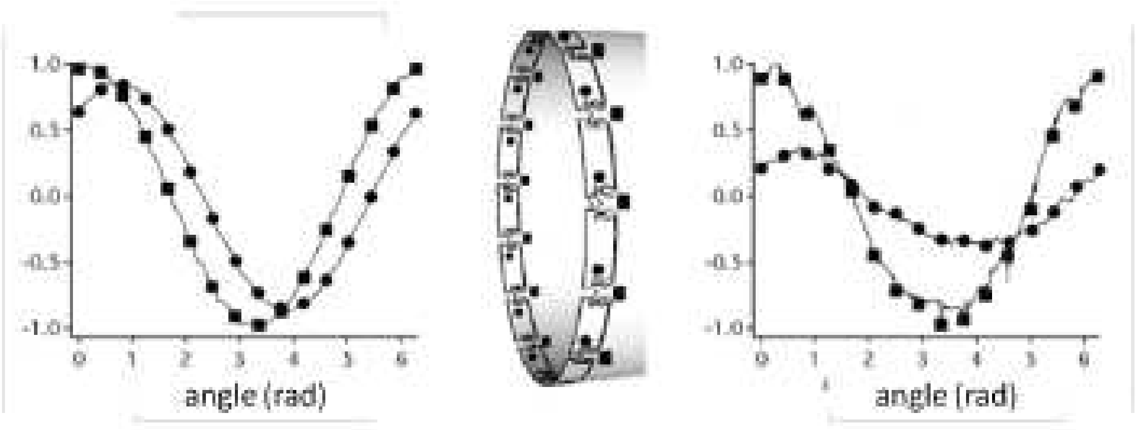
What's wrong ?

- Remember the full LES oscillates at 740 Hz
- This corresponds to a critical value for the time delay of T/2 = 0.67 ms, very close to the prescribed time delay
- From the LES, the flow rate and HR are indeed in phase opposition



Back to the stability of the 1A mode

- Instead of imposing the time delay in the FTF, let's impose the phase shift between flow rate and HR
- The 1A mode is now found unstable by the Helmholtz solver, consistently with LES and experiment
- The mode shape is also in better agreement



Helmholtz solver **LES solver**

THANK YOU

More details, slides, papers, …

http://www.math.univ-montp2.fr/~nicoud/